UDC 519.957

SEISMIC EXCITATION MODEL OF HALF-SPACE PROPAGATION OF RAYLEIGH WAVES

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This article presents a scientific study of seismic oscillations and Rayleigh wave propagation models. The research details how Rayleigh waves propagate in a semi-infinite elastic medium, the types of motions they create on Earth's surface, and how their amplitude decreases with depth. In the first section, the study examines Rayleigh waves and their mathematical representations, illustrating how these waves form and propagate in a semi-infinite medium. In addition, the relationships between wave amplitude and other parameters are expressed by mathematical equations. The following sections deal with the problem of defining the elastic properties of the medium taking boundary conditions into account. The study provides an analysis of strains and stress tensors, discusses their role in wave propagation, and describes in detail the components of stress and strain at each point. To solve problems with geometric symmetry, the Boundary Element Method (BEM) is used. Using the Morrow Point Dam model as an example, the study explains how this approach helps reduce computational effort by taking symmetry planes into account. It also describes the balance of hydrodynamic pressure and normal stresses at the interface between water and solid media. This article serves as a valuable resource for understanding the mathematical and physical principles, computational approaches, and boundary conditions in wave propagation that are critical to geophysical applications. Finally, the study highlights how the amplitude of Rayleigh waves changes with depth in a semi-infinite medium and discusses the importance of elastic constants in controlling these changes. This research provides essential theoretical insights useful for geological and engineering practices.

Keywords: seismic vibrations, surface waves, computational model, elastic medium, stress and strain tensors, geometric symmetry, hydrodynamic pressure, incident and diffracted fields.

Citation: Kholmurodov A.E., Matanov M.Ch. 2024. Seismic excitation model of half-space propagation of Rayleigh waves. *Problems of Computational and Applied Mathematics.* 6(62): 45-56.

1 Introduction

After analyzing the wave propagation mechanism P and S its various peculiarities, this section deals with the study of Rayleigh waves. Rayleigh waves are surface waves that produce retrograde elliptical motion of the ground. They are slower waves than bulk waves and their propagation speed is almost 70% of the propagation speed of waves S. As seen below, a plane Rayleigh wave propagating in a viscoelastic half-space itself confirms the governing equation of the problem. To verify this statement, we assume the displacement field caused by a wave of this type propagating in the positive direction of the axis x_2

with a speed c and a wave number $k = \omega/c$.

$$\begin{aligned} u_1 &= 0, \\ u_2 &= Ae^{bx_3}e^{ik(ct-x_2)}, \\ u_3 &= Be^{bx_3}e^{ik(ct-x_2)}. \end{aligned} \tag{1}$$

The non-zero components of the displacement field defined by expressions (1) are the result of the product of two exponential functions. The second of them $e^{ik(ct-x_2)}$ represents a travelling wave that propagates with speed c according to the positive direction of the axis x_2. The first e^{bx_3}, taking into account the direction of the axes with which we worked (note that the value of the coordinate $x_3$is always negative since the half-space has been defined), for b positive values of leads to a negative exponent, which implies that the amplitude of the wave decreases with depth, a characteristic phenomenon of this type of waves. Figure 1.1 shows the motion experienced by a soil particle when a wave of the type analyzed propagates.

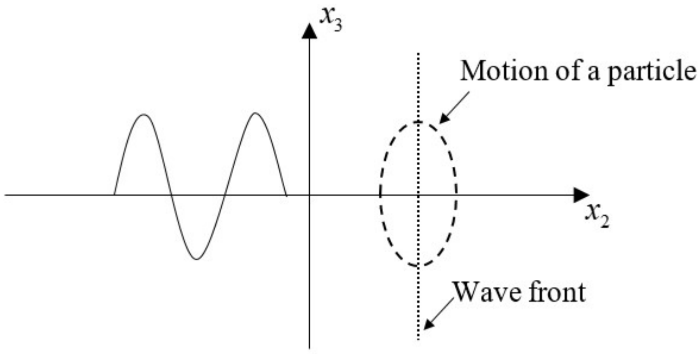

Figure 1 Propagation and motion of a particle caused by a Raleigh wave.

Substituting the displacements given by expressions (1.1) for the two non-zero components of the displacement (u_2, u_3)in the Navier equation:

$$\begin{gathered} \mu\left[-k^2Ae^{bx_3}e^{ik(ct-x_2)} + b^2Ae^{bx_3}e^{ik(ct-x_2)}\right] + (\lambda+\mu) \\ \left[-k^2Ae^{bx_3}e^{ik(ct-x_2)} + ikbBe^{bx_3}e^{ik(ct-x_2)}\right] = -\rho\omega^2Ae^{bx_3}e^{ik(ct-x_2)} \\ \mu\left[-k^2Be^{bx_3}e^{ik(ct-x_2)} + b^2Be^{bx_3}e^{ik(ct-x_2)}\right] + (\lambda+\mu) \\ \left[ikbAe^{bx_3}e^{ik(ct-x_2)} + b^2Be^{bx_3}e^{ik(ct-x_2)}\right] = -\rho\omega^2Be^{bx_3}e^{ik(ct-x_2)}. \end{gathered} \tag{2}$$

Rearranged and taking out a common factor A, B the above equations can be written as:

$$\begin{gathered} [b^2\mu - k^2(\lambda+2\mu) + \rho\omega^2]\,A + ikb(\lambda+\mu)B = 0, \\ ikb(\lambda+\mu)A + [b^2(\lambda+2\mu) - k^2\mu + \rho\omega^2]\,B = 0. \end{gathered} \tag{3}$$

So that the system of two equations and two unknowns given by (3) has a non-trivial solution, i.e. for A and B to be different from zero, the determinant of the system must be zero. So it is an eigenvalue problem that determines values of b that lead to a solution other than the trivial one.

$$\begin{vmatrix} b^2\mu - k^2(\lambda+2\mu) + \rho\omega^2 & ikb(\lambda+\mu) \\ ikb(\lambda+\mu) & b^2(\lambda+2\mu) - k^2\mu + \rho\omega^2 \end{vmatrix} = 0.$$

Therefore:

$$\left[b^2\mu - k^2(\lambda+2\mu) + \rho\omega^2\right]\left[b^2(\lambda+2\mu) - k^2\mu + \rho\omega^2\right] - \left[ikb(\lambda+\mu)\right]^2 = 0. \tag{4}$$

Dividing the above expression by ρ and taking into account the following identities:

$$\frac{\mu}{\rho} = c_s^2 \quad \left(\frac{\lambda+2\mu}{\rho}\right) = c_p^2 \quad \left(\frac{\lambda+\mu}{\rho}\right) = c_p^2 - c_s^2 \quad \omega^2 = k^2c^2.$$

Equation (4) becomes:

$$\begin{gathered} b^4(c_s^2c_p^2) + b^2\left[k^2c_s^2(c^2-c_s^2) + k^2c_p^2(c^2-c_p^2) + k^2(c_p^2-c_s^2)^2\right] + \\ +k^4(c^2-c_p^2)(c^2-c_s^2) = 0. \end{gathered} \tag{5}$$

The four solutions of equation (5) are:

$$\begin{aligned} b_1^2 &= k^2\left(1-\frac{c^2}{c_s^2}\right) \;\rightarrow\; b_1 = \pm k\sqrt{\left(1-\frac{c^2}{c_s^2}\right)}, \\ b_2^2 &= k^2\left(1-\frac{c^2}{c_p^2}\right) \;\rightarrow\; b_2 = \pm k\sqrt{\left(1-\frac{c^2}{c_p^2}\right)}. \end{aligned} \tag{6}$$

Taking the positive roots (remember the need for the parameter b to be positive so that the amplitude decreases with depth and the known physical reality is fulfilled) and substituting them in the first of the equations (3) we have:

$$b = b_1 \;\rightarrow\; \left[k^2(1-\frac{c^2}{c_s^2})c_s^2 + k^2(c^2-c_s^2)\right] + ik\left[k(1-\frac{c^2}{c_s^2})^{\frac{1}{2}}\right](c_p^2-c_s^2)\left(\frac{B}{A}\right) = 0. \tag{7}$$

Which simplified leads to a relationship between the amplitudes of the two waves involved.

$$\left(\frac{B}{A}\right)_1 = -\frac{ik}{k(1-\frac{c^2}{c_s^2})^{\frac{1}{2}}} = -\frac{ik}{b_1} \tag{8}$$

$$b = b_2 \;\rightarrow\; \left[k^2(1-\frac{c^2}{c_p^2})c_s^2 + k^2(c^2-c_p^2)\right] + ik\left[k(1-\frac{c^2}{c_p^2})^{\frac{1}{2}}\right](c_p^2-c_s^2)\left(\frac{B}{A}\right) = 0. \tag{9}$$

Which operating in an analogous manner leads to:

$$\left(\frac{B}{A}\right)_2 = \frac{k(1-\frac{c^2}{c_p^2})^{\frac{1}{2}}}{ik} = \frac{b_2}{ik}. \tag{10}$$

Given the development, we can conclude that to solve the problem there must be a relationship between the amplitudes A and B of the waves that depends on the values of b. In this way, the displacement field for this type of waves that satisfies the governing equation is the one defined by (11).

$$\begin{aligned} u_1 &= 0 \\ u_2 &= A_1e^{b_1x_3}e^{ik(ct-x_2)} + A_2e^{b_2x_3}e^{ik(ct-x_2)} \\ u_3 &= -\tfrac{ik}{b_1}A_1e^{b_1x_3}e^{ik(ct-x_2)} + \tfrac{b_2}{ik}A_2e^{b_2x_3}e^{ik(ct-x_2)}. \end{aligned} \tag{11}$$

2 Imposition of boundary conditions

To fully define the displacements, it is necessary to determine the value of the amplitudes A_1 and A_2 the wave number k. To do this, we will apply the boundary conditions corresponding to the half-space $x_3 = 0$. There are no tensions on the free surface.

$$x_3 = 0 \quad \to \quad \begin{cases} \sigma_{23} = 0, \\ \sigma_{33} = 0. \end{cases} \tag{12}$$

Therefore:

$$\begin{gathered} \sigma_{23} = \mu \left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2} \right) = 0, \\ \sigma_{33} = 2\mu \frac{\partial u_3}{\partial x_3} + \lambda \left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} \right) = 0. \end{gathered} \tag{13}$$

Substituting the derivatives of the displacement field given by expressions (11) in equations (13) we arrive at:

$$x_3 = 0 \quad \to \quad \begin{cases} b_1 \left(1 + \frac{k^2}{b_1^2}\right) A_1 + 2b_2 A_2 = 0; \\ 2\mu A_1 + \left[2\mu \frac{b_2^2}{k^2} - \lambda \left(1 - \frac{b_2^2}{k^2}\right)\right] A_2 = 0. \end{cases} \tag{14}$$

Calling $\frac{c^2}{c_s^2} = \gamma_s$ and $\frac{c^2}{c_p^2} = \gamma_p$ it is possible to write:

$$\frac{k^2}{b_1^2} = \frac{1}{1 - \frac{c^2}{c_s^2}} = \frac{1}{1 - \gamma_s} \qquad \frac{b_2^2}{k^2} = 1 - \frac{c^2}{c_p^2} = 1 - \gamma_p. \tag{15}$$

Expressions that substituted in (14), taking into account the expression of b_1 and of b_2 and after making a series of simplifications lead to the system of equations:

$$\begin{gathered} (2 - \gamma_s)A_1 + 2(1 - \gamma_p)^{\frac{1}{2}}(1 - \gamma_s)^{\frac{1}{2}} A_2 = 0, \\ 2A_1 + (2 - \gamma_s)A_2 = 0. \end{gathered} \tag{16}$$

In order for A_1 and A_2 to have a value other than trivial, the determinant of the system must be zero. This is also an eigenvalue problem, which, as we will see below, the values of γ_s and of γ_p since both are related to each other via the elastic constants. Therefore:

$$\begin{vmatrix} (2 - \gamma_s) & 2(1 - \gamma_p)^{\frac{1}{2}}(1 - \gamma_s)^{\frac{1}{2}} \\ 2 & (2 - \gamma_s) \end{vmatrix} = 0. \tag{17}$$

Whose characteristic equation, taking into account the relationship $\gamma_p = \frac{\mu}{\lambda + 2\mu}\gamma_s$, can be written as a function of one of the variables as:

$$(2 - \gamma_s)^2 - 4\left(1 - \frac{\mu}{\lambda + 2\mu}\gamma_s\right)^{\frac{1}{2}} (1 - \gamma_s)^{\frac{1}{2}} = 0. \tag{18}$$

The solution of (18) leads to the value of γ_s and therefore to the propagation speed c if the properties of the medium are known.

On the other hand, through the second of the equations (16) it is possible to find a relationship between A_1 and A_2:

$$A_2 = -\left(\frac{2}{2 - \gamma_s}\right) A_1. \tag{19}$$

If we introduce this relationship in the expression of the displacement field given by the expressions (11) it remains as:

$$\begin{aligned} u_1 &= 0 \\ u_2 &= A_1\left(e^{b_1x_3} + \frac{2}{2-\gamma_s}e^{b_2x_3}\right)e^{ik(ct-x_2)} \\ u_3 &= A_1\left(-\frac{ik}{b_1}e^{b_1x_3} + \frac{b_2}{ik}\frac{2}{2-\gamma_s}e^{b_2x_3}\right)e^{ik(ct-x_2)}. \end{aligned} \tag{20}$$

3 Strain and stress tensors

Once the displacement field expressions are known, it is possible to determine the strain and stress tensors. As usual, the deformation tensor is first calculated using the compatibility equations and then the expression for the stress tensor at each point in the half-space determined using the behavior law. Taking into account the displacements determined in the previous section, Equations (20), the deformation tensor looks like this:

$$\varepsilon_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \varepsilon_{22} & \varepsilon_{23} \\ 0 & \varepsilon_{32} & \varepsilon_{33} \end{pmatrix}. \tag{21}$$

Whose non-zero components are the following:

$$\begin{aligned} \varepsilon_{22} &= \frac{\partial u_2}{\partial x_2} = -ik\left(A_1e^{b_1x_3} + A_2e^{b_2x_3}\right)e^{-ikx_2}, \\ \varepsilon_{33} &= \frac{\partial u_3}{\partial x_3} = -ik\left(A_1e^{b_1x_3} - \frac{b_2^2}{ik}A_2e^{b_2x_3}\right)e^{-ikx_2}, \\ \varepsilon_{23} = \varepsilon_{32} &= \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right) = \frac{1}{2}\left(\left(b_1 + \frac{k^2}{b_1}\right)A_1e^{b_1x_3} + 2b_2A_2e^{b_2x_3}\right)e^{-ikx_2}. \end{aligned} \tag{22}$$

Once the strain tensor is determined, it is possible to determine the components of the stress tensor using the material behavior law:

$$\sigma_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_{22} & \sigma_{23} \\ 0 & \sigma_{32} & \sigma_{33} \end{pmatrix}. \tag{23}$$

where the non-zero components are:

$$\begin{aligned} \sigma_{11} &= 2\mu\varepsilon_{11} + \lambda\varepsilon_{kk} = -\lambda\left(ik + \frac{b_2^2}{ik}\right)A_2e^{b_2x_3}e^{-ikx_2}, \\ \sigma_{22} &= 2\mu\varepsilon_{22} + \lambda\varepsilon_{kk} = \left[-2\mu ikA_1e^{b_1x_3} - \left(\lambda\left(ik + \frac{b_2^2}{ik}\right) + 2\mu ik\right)A_2e^{b_2x_3}\right]e^{-ikx_2}, \\ \sigma_{33} &= 2\mu\varepsilon_{33} + \lambda\varepsilon_{kk} = \left[2\mu ikA_1e^{b_1x_3} - \left((\lambda+2\mu)\left(\frac{b_2^2}{ik}\right) + 2\mu ik\right)A_2e^{b_2x_3}\right]e^{-ikx_2} \\ \sigma_{23} = \sigma_{32} &= 2\mu\varepsilon_{23} = \mu\left[\left(b_1 + \frac{k^2}{b_1}\right)A_1e^{b_1x_3} - 2bA_2e^{b_2x_3}\right]e^{-ikx_2}. \end{aligned} \tag{24}$$

It is possible to find a parallel between the expressions obtained for the displacement field caused by the propagation of a Rayleigh wave and the expression (*) used to calculate the field caused by waves P and S in previous sections. If we compare the general expression of the field given by (*), concretized for the problem addressed, with those obtained for the displacement field of a Rayleigh wave, equations (20), in terms of the vectors, the direction cosines of the following include the displacements and the propagation direction vectors:

$$d^{(0)} = \left[0,\ 1,\ -\frac{ik}{b_1}\right], \qquad d^{(1)} = \left[0,\ 1,\ -\frac{b_2}{ik}\right], \tag{25}$$

$$s^{(0)} = \left[0,\ 1,\ -\frac{ib_1}{k}\right], \qquad s^{(1)} = \left[0,\ 1,\ \frac{ib_2}{k}\right]. \tag{26}$$

The complex nature of the components in the direction is noticeable x_3.

4 Extension of two-dimensional expressions to the general problem in 3 dimensions

The formulation presented so far allows us to consider a wave contained in the plane with a generic incidence x_2x_3. However, it does not reflect the possible incidence included in another level. The goal of this section is to implement this possibility.

Figure 2 shows a representation of the axis system used and the relationship between the axes previously used to solve the problem, henceforth, called $(\tilde{x}_2\tilde{x}_3)$ and the new generic axes (x_2x_3), $(x_3 = \tilde{x}_3)$:

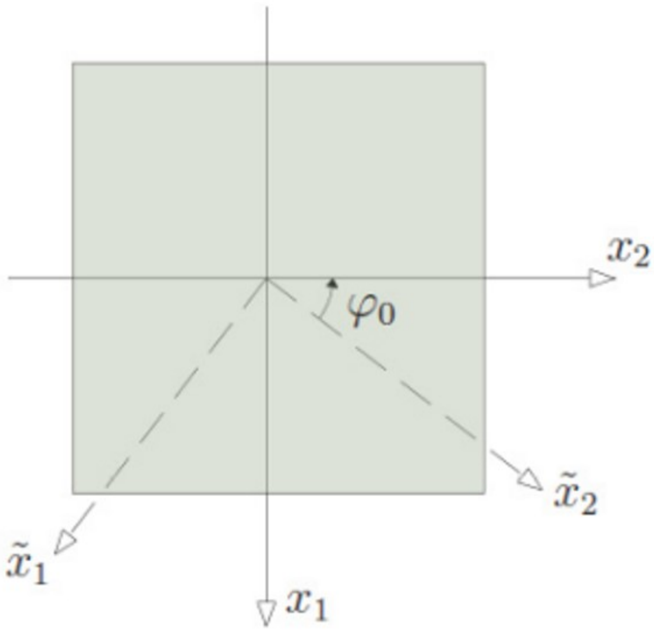

Figure 2 Relationship between the $\tilde{x}_2\tilde{x}_3$ and axes x_2x_3.

By defining a set of unit vectors in the direction of the three Cartesian axes of the initial problem $(\tilde{i}_1, \tilde{i}_2, \tilde{i}_3)$and another in the direction of the new ones (i_1, i_2, i_3), it can be shown that the following relationship exists between them: φ

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} \cos(\varphi_0) & \sin(\varphi_0) & 0 \\ -\sin(\varphi_0) & \cos(\varphi_0) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{i}_1 \\ \tilde{i}_2 \\ \tilde{i}_3 \end{bmatrix}. \tag{27}$$

The matrix that relates the set of unit vectors of the initial axis system with that of the system $x_1x_2x_3$ is called the rotation matrix and will be denoted from now on by the letter R.

Although the expressions are of completely general application, the parameters relative to the wave will be used as an example SV. This is done for illustrative purposes only, and the expressions and conclusions are completely general. In this sense, the displacement field in the initial axis system $(\tilde{x}_1\tilde{x}_2\tilde{x}_3)$ can be expressed as:

$$\tilde{u} = \tilde{d}^{(0)} A_{sv}^{inc} e^{-ik_s(\tilde{s}^{(0)}\cdot\tilde{r})} + \tilde{d}^{(1)} A_{sv}^{ref} e^{-ik_s(\tilde{s}^{(1)}\cdot\tilde{r})} + \tilde{d}^{(2)} A_{p}^{ref} e^{-ik_p(\tilde{s}^{(2)}\cdot\tilde{r})}. \tag{28}$$

Whose propagation and displacement vectors $\tilde{s}$ are $\tilde{d}$: θ

$$\begin{aligned}
\tilde{s}^{(0)} &= \begin{bmatrix} 0 \\ \cos(\theta_0) \\ \sin(\theta_0) \end{bmatrix}, & \tilde{d}^{(0)} &= \begin{bmatrix} 0 \\ \sin(\theta_0) \\ -\cos(\theta_0) \end{bmatrix}, \\
\tilde{s}^{(1)} &= \begin{bmatrix} 0 \\ \cos(\theta_1) \\ \sin(\theta_1) \end{bmatrix}, & \tilde{d}^{(1)} &= \begin{bmatrix} 0 \\ -\sin(\theta_1) \\ -\cos(\theta_1) \end{bmatrix}, \\
\tilde{s}^{(2)} &= \begin{bmatrix} 0 \\ \cos(\theta_2) \\ -\sin(\theta_2) \end{bmatrix}, & \tilde{d}^{(2)} &= \begin{bmatrix} 0 \\ \cos(\theta_2) \\ -\sin(\theta_2) \end{bmatrix}.
\end{aligned} \tag{29}$$

Premultiplying the displacement field expression by R we obtain:

$$R\tilde{u} = R\tilde{d}^{(0)} A_{sv}^{inc} e^{-ik_s(\tilde{s}^{(0)}\cdot\tilde{r})} + R\tilde{d}^{(1)} A_{sv}^{ref} e^{-ik_s(\tilde{s}^{(1)}\cdot\tilde{r})} + R\tilde{d}^{(2)} A_{p}^{ref} e^{-ik_p(\tilde{s}^{(2)}\cdot\tilde{r})}. \tag{30}$$

The scalar product expression $\tilde{s}^{(j)} \cdot \tilde{r}$ can be expressed as:

$$\tilde{s}^{(j)} \cdot \tilde{r} = \left(R^{-1}s^{(j)}\right)^T R^{-1}r = \left[s^{(j)}\right]^T \left(R^{-1}\right)^T R^{-1}r = \left[s^{(j)}\right]^T RR^{-1}r = s^{(j)}r. \tag{31}$$

According to this equality, it can be established that the product of the matrix R by the displacement field takes the value:

$$R\tilde{u} = R\tilde{d}^{(0)} A_{sv}^{inc} e^{-ik_s(s^{(0)}\cdot r)} + R\tilde{d}^{(1)} A_{sv}^{ref} e^{-ik_s(s^{(1)}\cdot r)} + R\tilde{d}^{(2)} A_{p}^{ref} e^{-ik_p(s^{(2)}\cdot r)}. \tag{32}$$

Also taking into account the following equalities:

$$\begin{aligned}
&u = R\tilde{u}, \\
&s^{(0)} = R\tilde{s}^{(0)} \quad d^{(0)} = R\tilde{d}^{(0)}, \\
&s^{(1)} = R\tilde{s}^{(1)} \quad d^{(1)} = R\tilde{d}^{(1)}, \\
&s^{(2)} = R\tilde{s}^{(2)} \quad d^{(2)} = R\tilde{d}^{(2)}.
\end{aligned} \tag{33}$$

The displacement field is finally expressed as:

$$u = d^{(0)} A_{sv}^{inc} e^{-ik_s(s^{(0)}\cdot r)} + d^{(1)} A_{sv}^{ref} e^{-ik_s(s^{(1)}\cdot r)} + d^{(2)} A_{p}^{ref} e^{-ik_p(s^{(2)}\cdot r)}. \tag{34}$$

Thus, the problem can be posed in a way analogous to that performed for the plane x_2x_3 with the exception that the vectors s and d are given by the expressions (33).

5 Seismic excitation model. Incorporation of the incident field equations into a coupled Boundary Element model.

In the models that are intended to be solved, the seismic excitation has been implemented as a field of plane harmonic waves in the ground that affects the reservoir location area from a distant point. As a consequence of the presence of the canyon, the dam and the reservoir, the field studied for each type of wave in the previous sections, which we have called the incident field (u_I^s), is distorted. We can consider the displacement field in the ground as the superposition of the displacement fields of two problems (figure 3). The first corresponds to that caused by the train of incident waves on the uniform half-space (u_I^s) whose analytical expression is that obtained previously. The second represents the field diffracted by the presence of the canyon-dam-reservoir system (u_D^s). Therefore the total field (u_T^s) in the ground will be the sum of both ($u_I^s = u_I^s + u_D^s$). In the dam, in the impounded water and in the porous sediment, however, there is only a diffracted field, the total field being equal to this $\left(u_T^p = u_D^p,\;\; u_T^a = u_D^a,\;\; u_T^{sed} = u_D^{sed}\right)$.

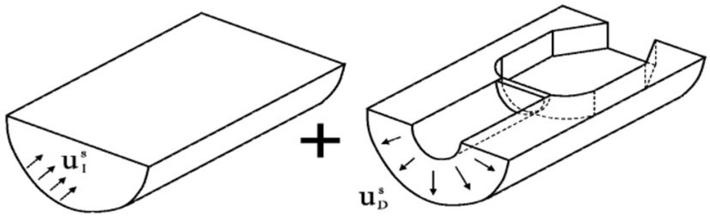

Figure 3 Seismic excitation model. Total ground field as superposition of the incident u_I^s and diffracted fields u_D^s.

Since the incident field is explicitly known, the problem is to calculate the diffracted field evaluation by means of the BEM. The system of boundary element equations proposed for the diffracted field in the two solid regions (soil and dam), in the fluid region (water) and in the porous region (sediment) leads to:

$$\begin{aligned} H^s\, u_D^s &= G^s\, t_D^s, \\ H^p\, u_D^p &= G^p\, t_D^p, \\ H^a\, p_D^a &= G^a \left(\frac{\partial p}{\partial n}\right)_T^a, \\ H^{sed}\, u_D^{sed} &= G^{sed}\, t_D^{sed}. \end{aligned} \tag{35}$$

For illustrative purposes, one of the boundary element networks of the system under discussion is shown in Figure 4. The aim is to solve a symmetrical problem (with respect to the vertical plane containing the long axis of the canyon) so that only half of the complete geometry is displayed. Although the free surface of the ground extends to infinity, the boundary element network only extends to a certain distance from the dam. This does not introduce significant errors since equations (35) are written in terms of the diffracted field that satisfies the radiation conditions. To minimize the error caused by these ground shortenings, it is necessary to determine, through a series of numerical tests, the distance from the dam that ensures the attenuation of the diffracted field, thereby defining the free surface distance to be discretized. With all that has been said about the total field in each region, equations (35) can be written in terms of the total field of displacements

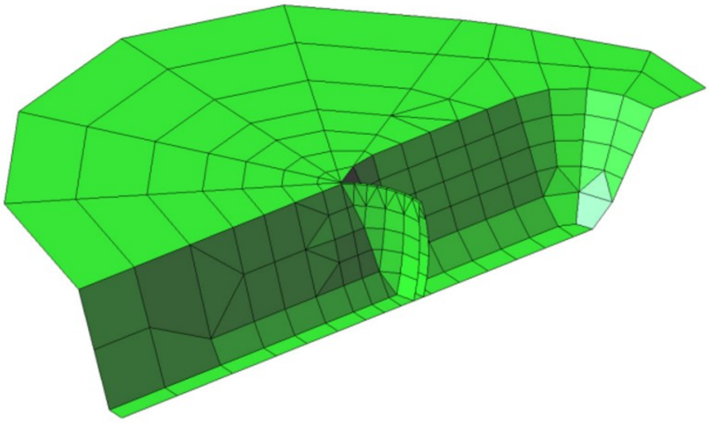

Figure 4 Boundary element model for Morrow Point Dam.

and stresses in the soil, the dam, the water and sediment as follows:

$$\begin{aligned} H^s\, u_T^s \,-\, G^s\, t_T^s &= H^s\, u_I^s \,-\, G^s\, t_I^s, \\ H^p\, u_T^p \,&=\, G^p\, t_T^p, \\ H^a\, p_T^a \,&=\, G^a \left(\frac{\partial p}{\partial n}\right)_T^a, \\ H^{sed}\, u_T^{sed} \,-\, G^{sed}\, t_T^{sed} &= 0. \end{aligned} \tag{36}$$

The boundary conditions and interface conditions are applied to these equations, expressed as the total field. Thus, the state of zero tension is exerted on the free soil surface and on the dam walls that do not come into contact with water. Stress balance and continuity of displacements are prescribed in the elements of the dam-soil interface. At the interfaces between the water region and the solid region, the absence of tangential stress in the solid and the equality of normal stress and hydrodynamic pressure in the fluid are determined. The kinematic condition in this case is the equality of displacements in the direction normal to the interface.

6 Treatment and implementation of field equations in problems with geometric symmetry planes.

The code used to solve the problems posed allows, in cases where geometric symmetry is present, in order to reduce the number of degrees of freedom and therefore reduce the computing time, to discretize only the part necessary to define the geometry of the problem. For example, and as previously mentioned, in the proposed Morrow Point Dam boundary element model, there is a geometric plane of symmetry formed by the axes running toward the canyon closure on one side and the elevation of the dam on the other side, so that only half of the set needs to be discretized, see Figure 1.4.

Solving the problem requires solving two cases, one symmetrical and one antisymmetric, since there is no symmetry in the stress (note that except in certain vertical incidence cases, the wave does not impinge on the model symmetrically). In order to take advantage of geometric symmetry, it is necessary to adapt the expressions for the incident field developed in the previous sections to the specific case of symmetry. To do this, we start from the generic expression for the displacement field (equation (*)):

$$u_i = \sum_{j=0}^{n} d_i^j A_j e^{-ik_j(s^{(j)}\cdot r)}. \qquad (*)$$

Suppose we call the plane with respect to which geometric symmetry exists x_1x_2, the terms affected by the symmetry are those that contain references to the spatial coordinates, in this case, the exponential terms and in particular those related to the coordinate x_3. Developing the scalar product present in the exponent of the previous expression for wave$j$, we have:

$$e^{-ik_j(s^{(j)}\cdot r)} = e^{-ik_j(s_1^{(j)}\cdot x_1)}e^{-ik_j(s_2^{(j)}\cdot x_2)}e^{-ik_j(s_3^{(j)}\cdot x_3)}. \tag{37}$$

The last of these terms, the one affected by the symmetry conditions, can be written as:

$$\begin{gathered} e^{-ik_j(s_3^{(j)}\cdot x_3)} = \frac{1}{2}\left[e^{-ik_j(s_3^{(j)}\cdot x_3)} + e^{-ik_j(s_3^{(j)}\cdot x_3)}\right] + \frac{1}{2}\left[e^{-ik_j(s_3^{(j)}\cdot x_3)} - e^{-ik_j(s_3^{(j)}\cdot x_3)}\right] = \\ = \cos\left(k_j s_3^{(j)} x_3\right) + i\sin\left(k_j s_3^{(j)} x_3\right). \end{gathered} \tag{38}$$

In order to simplify the nomenclature, equation (38) can be written as:

$$e^{-ik_j(s_3^{(j)}\cdot x_3)} = ezc(j) + ezs(j). \tag{39}$$

where:

$$\begin{aligned} ezc(j) &= \cos\left(k_j s_3^{(j)} x_3\right), \\ ezs(j) &= i\sin\left(k_j s_3^{(j)} x_3\right). \end{aligned} \tag{40}$$

By analogy, the components not affected by symmetry ($x_1$and x_2) can be written as:

$$\begin{aligned} e^{-ik_j(s_1^{(j)}\cdot x_1)} &= ex(j), \\ e^{-ik_j(s_2^{(j)}\cdot x_2)} &= ey(j). \end{aligned} \tag{41}$$

By introducing expressions (39) and (41) the exponential of expression (*) becomes:

$$e^{-ik_j(s^{(j)}\cdot r)} = ex(j)ey(j)\left[ezc(j) + ezs(j)\right] = \underbrace{ex(j)ey(j)ezc(j)}_{Symmetric\ part} + \underbrace{ex(j)ey(j)ezs(j)}_{Antisymmetric\ part}. \tag{42}$$

Thus, the displacement field is expressed as a superposition of a symmetric and an antisymmetric problem:

$$u_i = \sum_{j=0}^{n} d_i^j A_j e^{-ik_j(s^{(j)}\cdot r)} = \sum_{j=0}^{n} d_i^j A_j ex(j)ey(j)ezc(j) + \sum_{j=0}^{n} d_i^j A_j ex(j)ey(j)ezs(j). \tag{43}$$

Once the displacement field is determined as the sum of two problems, it is easy to obtain the expressions for deformations and stresses using the compatibility equations and the law of behavior of the medium with analogous decomposition.

7 Conclusion

In summary, this study successfully developed a seismic excitation model to analyze the propagation of Rayleigh waves in a semi-infinite elastic medium. By studying the wave's displacement field, boundary conditions and stress-strain tensors, the research demonstrates the complex behavior of Rayleigh waves, including their amplitude decay with depth. The application of the Boundary Element Method (BEM), particularly using the example of Morrow Point Dam, highlights the utility of geometric symmetry in simplifying computational requirements. Overall, this research provides fundamental insights into seismic wave behavior in elastic media that are critical for applications in geophysics and earthquake-resistant design.

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Received November 20, 2024

УДК 519.957

МОДЕЛЬ СЕЙСМИЧЕСКОГО ВОЗБУЖДЕНИЯ ПОЛУПРОСТРАНСТВЕННОГО РАСПРОСТРАНЕНИЯ ВОЛН РЭЛЕЯ

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В этой статье представлено научное исследование сейсмических колебаний и моделей распространения волн Рэлея. В исследовании подробно описывается, как волны Рэлея распространяются в полубесконечной упругой среде, типы движений, которые они создают на поверхности Земли, и как их амплитуда уменьшается с глубиной. В первом разделе исследования рассматриваются волны Рэлея и их математические представления, иллюстрирующие, как эти волны формируются и распространяются в полубесконечной среде. Кроме того, соотношения между амплитудой волны и другими параметрами выражаются математическими уравнениями. В следующих разделах рассматривается проблема определения упругих свойств среды с учетом граничных условий. В исследовании представлен анализ деформаций и тензоров напряжений, обсуждается их роль в распространении волн и подробно описываются компоненты напряжения и деформации в каждой точке. Для решения задач с геометрической симметрией используется метод граничных элементов (BEM). Используя в качестве примера модель плотины Морроу-Пойнт, исследование объясняет, как этот подход помогает сократить вычислительные затраты за счет учета плоскостей симметрии. В нем также описывается баланс гидродинамического давления и нормальных напряжений на границе раздела между водой и твердыми средами. Эта статья служит ценным ресурсом для понимания математических и физических принципов, вычислительных подходов и граничных условий в распространении волн, которые имеют решающее значение для геофизических приложений. Наконец, в исследовании подчеркивается, как амплитуда волн Рэлея изменяется с глубиной в полубесконечной среде, и обсуждается важность упругих констант в управлении этими изменениями. Это исследование дает важные теоретические идеи, полезные для геологической и инженерной практики.

Ключевые слова: сейсмические колебания, поверхностные волны, вычислительная модель, упругая среда, тензоры напряжений и деформаций, геометрическая симметрия, гидродинамическое давление, падающие и дифрагированные поля.

Цитирование: *Холмуродов А.Э., Матанов М.Ч.* Модель сейсмического возбуждения полупространственного распространения волн Рэлея // Проблемы вычислительной и прикладной математики. – 2024. – № 6(62). – С. 45-56.

ПРОБЛЕМЫ ВЫЧИСЛИТЕЛЬНОЙ И ПРИКЛАДНОЙ МАТЕМАТИКИ

№ 6(62) 2024

Журнал основан в 2015 году.
Издается 6 раз в год.

Учредитель:
Научно-исследовательский институт развития цифровых технологий и искусственного интеллекта.

Журнал зарегистрирован в Агентстве информации и массовых коммуникаций при Администрации Президента Республики Узбекистан.
Регистрационное свидетельство №0856 от 5 августа 2015 года.

ISSN 2181-8460, eISSN 2181-046X

Адрес редакции:
100125, г. Ташкент, м-в. Буз-2, 17А.
Тел.: +(998) 712-319-253, 712-319-249.
Э-почта: `journals@airi.uz`.
Веб-сайт: `https://journals.airi.uz`.

Дизайн и вёрстка:
Шарипов Х.Д.

Отпечатано в типографии НИИ РЦТИИ.
Подписано в печать 27.12.2024 г.
Формат 60x84 1/8. Заказ №8. Тираж 100 экз.

PROBLEMS OF COMPUTATIONAL AND APPLIED MATHEMATICS

No. 6(62) 2024

The journal was established in 2015.
6 issues are published per year.

Founder:
Digital Technologies and Artificial Intelligence Development Research Institute.

The journal is registered by Agency of Information and Mass Communications under the Administration of the President of the Republic of Uzbekistan.
The registration certificate No. 0856 of 5 August 2015.

ISSN 2181-8460, eISSN 2181-046X

Address:
100125, Tashkent, Buz-2, 17A.
Tel.: +(998) 712-319-253, 712-319-249.
E-mail: journals@airi.uz.
Web-site: https://journals.airi.uz.

Layout design:
Sharipov Kh.D.

DTAIDRI printing office.
Signed for print 27.12.2024
Format 60x84 1/8. Order No. 8. Printed copies 100.

Содержание

Contents