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NEW COUPLED THERMOELASTICITY BOUNDARY-VALUE PROBLEMS IN STRAINS

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Using the Duhamel-Neumann relationship and the equations of motion, the Saint-Venant compatibility conditions, are written as a system of six dynamic equations with respect to strains. It is shown that, unlike the Beltrami-Michell equations, these equations are independent and can be considered as dynamic equations of thermoelasticity.Considering these equations together with the heat influx equations, a coupled thermoelasticity problem in strains is formulated. It is shown, also that replacing, the first three equations in formulated coupled problem, with the motion equations allows us to set an alternative coupled problem in strains. Using the two proposed formulations the coupled thermoelastic problem on a rectangular plate is numerically solved. The validity of the formulated two boundary value problems of thermoelasticity is justified by comparing their numerical, obtained by the variable direction method and recurrence relations, as well as solving a similar related problem regarding displacements.

Keywords: Saint-Venant compatibility condition, finite-difference method, explicit and implicit schemes, variable direction method.

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1 Introduction

Study of the stress-strain state of solids under the thermomechanical factors, is an actual problems for mechanical engineering. Usually, for study the thermoelastic problems the Duhamel–Neumann's constitutive relation is used. Typically, when solving thermoelastic boundary value problems, temperature fields are usually considered to be known as a solution to the heat flow equation. Joining the flow equation to motion equations and Duhamel–Neumann's constitutive relations one can receive coupled boundary value problem of thermoelasticity with respect to displacements and temperature.

In the coupled boundary value problem of thermoelasticity if we neglect the inertial terms, the motion and heat flow equations can be considered independently of each other, and then the boundary problem becomes uncoupled [11, 16]. Usually coupled boundary problems formulate regarding the displacement and temperature. However, formulation coupled problems for strains and temperature allows to describe the process deformation more adequate.

Boundary value problems of thermoelasticity regarding to strains can be formulated within the framework of the Saint-Venant compatibility condition [1, 2, 7]. The strain compatibility condition using the Duhamel–Neumann relation and the motion equations can be reduced to system of six interconnected differential equations regarding the strains and temperature [4].

It is known that of the six compatibility conditions, only three are independent [31,8,1]. In the case of plane problems, the compatibility condition consists of one relation, which, together with two equilibrium equations, constitutes a boundary value problem [32]. In Pobedry's works, the compatibility conditions of strains were reduced to six independent equations [3].

When formulating boundary value problems regarding strains and stresses, usually the number of boundary conditions is less than the number of unknowns. In order to overcome this obstacle, by Pobedri [2, 5] was proposed to consider the equation of motion as a boundary condition on the boundary of the considered domain.

Despite the progress achieved in the field of numerical methods, usually the literature is limited to solving plane problems of thermoelasticity by reducing it to a biharmonic equation with respect to the Airy stress function [14, 26].

It can be noted the Filonenko-Borodich problem on the compression of the parallelepiped under the domed load by the variation method [17, 18]. The static problem in strains is considered in [12, 13, 19]. In [20, 21], the coupled thermoelasticity problems was solved numerically using the finite difference method. Dynamic boundary value problems in Konovalov's works [22, 23].

The formulation and numerical solution of the coupled thermoelasticity problems regarding the strains and temperature are the main goal of the paper. Finite-difference equations are constructed in the form of explicit and implicit schemes.The plate problem of thermoelasticity with respect to strains is numerically solved. The validity of the formulated coupled problems and received results are shown solving by various methods and comparison with other results of a rectangular plate formulated in displacements.

2 Statement of the coupled thermoelastic problem in strains

It is known [15, 16] that the coupled thermoelasticity problem for isotropic bodies consists of the motion equation

$$\sigma_{ij,j} + \rho X_i = \rho \ddot{u}_i, \tag{1}$$

Duhamel-Neumann relations [32]

$$\sigma_{ij} = \lambda\theta\delta_{ij} + 2\mu\varepsilon_{ij} - \gamma\alpha(T - T_0)\delta_{ij}, \quad \gamma = 3\lambda + 2\mu, \tag{2}$$

Cauchy relations

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{3}$$

heat gain equation [15]

$$\lambda_0\theta_{,ii} - C_\varepsilon\dot{\theta} - T_0\gamma\dot{\varepsilon}_{ii} = -w, \tag{4}$$

and initial and boundary conditions

$$u_i|_{t=t_0} = \varphi_i, \frac{\partial u_i}{\partial t}|_{t=t_0} = \psi_i, T|_{t=t_0} = \tilde{T}, \tag{5}$$

$$u_i|_{\sum_1} = u_i^0, \ \sigma_{ij}n_j|_{\sum_2} = S_i, \ T|_{\sum} = T^0, \tag{6}$$

where σ_{ij} – a stress tensor, ε_{ij} – a strain tensor, u_i – the displacements, $\ddot{u}_i$ – the second derivative of displacements, T – the temperature, ρ – the density λ, μ – the elastic constants, θ – spherical part of the strain tensor, λ_0 – thermal conductivity coefficient, C_ε – the heat capacity coefficient, α – the thermal expansion coefficient, w – the heat source,

S_i – the surface load, n_i – the normal to the surface, Σ, X_i – the body forces, δ_{ij} – the Kronecker symbol.

The equation of motion (1) using relations (2) and (3) can be written relative to displacements [11, 12, 16], i.e.

$$\mu\nabla^2 u_i + (\lambda+\mu)\theta_{,i} - (3\lambda+2\mu)\alpha\frac{\partial T}{\partial x_i} + \rho X_i = \rho\ddot{u}_i, \tag{7}$$

where ∇^2–denotes the Laplace operator, $\theta = \varepsilon_{kk}$.

Differentiation equation (7) according to x_j i.e.

$$\mu\nabla^2 u_{i,j} + (\lambda+\mu)\theta_{,ij} - (3\lambda+2\mu)\alpha T_{,ij} + \rho X_{i,j} = \rho\ddot{u}_{i,j} \tag{8}$$

and swapping the indices i and j in (8)

$$\mu\nabla^2 u_{j,i} + (\lambda+\mu)\theta_{,ji} - (3\lambda+2\mu)\alpha T_{,ij} + \rho X_{j,i} = \rho\ddot{u}_{j,i} \tag{9}$$

and adding equations (8) and (9), we can find the following equation for deformations [19]

$$\mu\nabla^2 \varepsilon_{ij} + (\lambda+\mu)\theta_{,ij} - (3\lambda+2\mu)\alpha T_{,ij} + \frac{1}{2}\rho(X_{i,j}+X_{j,i}) = \rho\ddot{\varepsilon}_{ij}. \tag{10}$$

Note that the last equation can also be obtained from the compatibility condition using the Duhamel-Neumann relation and the equation of motion and is an analogue of the Saint-Venant compatibility condition. Therefore, following the works [15, 29], we can call it differential equations of strain compatibility.

Differential equations of strain compatibility (10) in combination with the heat flow equation [16]

$$\lambda_0\theta_{,ii} - C_\varepsilon\dot{\theta} - T_0\gamma\dot{\varepsilon}_{ii} = -w \tag{11}$$

with the corresponding initial

$$\varepsilon_{ij}|_{t=t_0} = f_i, \;\; \dot{\varepsilon}_{ij}|_{t=t_0} = \varphi_i, T|_{t=t_0} = \tilde{T} \tag{12}$$

and boundary conditions

$$\begin{gathered}[(\lambda\theta\delta_{ij} + 2\mu\varepsilon_{ij} - \gamma\alpha(T-T_0)\delta_{ij})n_j]\,|_{\sum_2} = S_i\,, \\ T|_{\sum} = T^0\end{gathered} \tag{13}$$

and additional conditions [2, 3] obtained from equations (1) and (2)

$$(\lambda\theta_{,i} + 2\mu\varepsilon_{ij\,,j} - \gamma\alpha T_{,i} + \rho X_i - \rho\ddot{u}_i\;)|_{\sum} = 0 \tag{14}$$

can be considered as a coupled thermoelasticity problem in strains (problem A).

For formulating the elasticity in stresses, may be consider the first or second group of three Beltrami-Michell equations in combination with three equilibrium equations [11, 15, 16]. According to [2], the closeness of the boundary problem requires three additional boundary conditions receiving from the equilibrium equations consider the boundary of a domain. If, in boundary problem (10-14), instead of the first three differential equations of (10), to consider following three motion equations expressed with respect to strains i.e.

$$\begin{gathered}\lambda\theta_{,11} + 2\mu\varepsilon_{1k,k1} - \gamma\alpha_{,11} + \rho X_{1,1} = \rho\ddot{\varepsilon}_{11}, \\ \lambda\theta_{,22} + 2\mu\varepsilon_{2k,k2} - \gamma\alpha_{,22} + \rho X_{2,2} = \rho\ddot{\varepsilon}_{22}\,, \\ \lambda\theta_{,33} + 2\mu\varepsilon_{3k,k3} - \gamma\alpha_{,33} + \rho X_{3,3} = \rho\ddot{\varepsilon}_{11}\end{gathered} \tag{15}$$

we receive a second form of the coupled thermoelasticity problem in strains **(problem B)**.

Note, equations (15), received from the equation of motion (1) differentiating it with respect to and, respectively and taking into account relation (2).

Thus, equations (15) together with the following three strain compatibility conditions i.e.

$$\mu\nabla^2\varepsilon_{ij} + (\lambda+\mu)\theta_{,ij} - \gamma\alpha T_{,ij} + \frac{1}{2}\rho(X_{i,j}+X_{j,i}) = \rho\ddot{\varepsilon}_{ij}, \;\; i \neq j \tag{16}$$

can be considered instead of equations (10) in problem A.

3 Plane thermoelastic problems in strains

We will consider the thermoelasticity problems of A and B, formulated in the previous paragraph, in the plane strain case. The problem A (10-14) in the absence of mass forces has the form

$$\begin{aligned}
(\lambda+2\mu)\frac{\partial^2\varepsilon_{11}}{\partial x^2} + \mu\frac{\partial^2\varepsilon_{11}}{\partial y^2} + (\lambda+\mu)\frac{\partial^2\varepsilon_{22}}{\partial x^2} - \gamma\frac{\partial^2 T}{\partial x^2} = \rho\frac{\partial^2\varepsilon_{11}}{\partial t^2},\\
(\lambda+2\mu)\frac{\partial^2\varepsilon_{22}}{\partial y^2} + \mu\frac{\partial^2\varepsilon_{22}}{\partial x^2} + (\lambda+\mu)\frac{\partial^2\varepsilon_{11}}{\partial y^2} - \gamma\frac{\partial^2 T}{\partial y^2} = \rho\frac{\partial^2\varepsilon_{22}}{\partial t^2},\\
\mu(\frac{\partial^2\varepsilon_{12}}{\partial x^2} + \frac{\partial^2\varepsilon_{12}}{\partial y^2}) + (\lambda+\mu)(\frac{\partial^2\varepsilon_{11}}{\partial x\partial y} + \frac{\partial^2\varepsilon_{22}}{\partial x\partial y}) - \gamma\frac{\partial^2 T}{\partial x\partial y} = \rho\frac{\partial^2\varepsilon_{12}}{\partial t^2},\\
\lambda_0(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) - C_\varepsilon\frac{\partial T}{\partial t} - \gamma T\frac{\partial(\varepsilon_{11}+\varepsilon_{22})}{\partial t} = 0.
\end{aligned} \tag{17}$$

In plane strain case, the coupled elasticity problem B takes the following form

$$\begin{aligned}
(\lambda+2\mu)\frac{\partial^2\varepsilon_{11}}{\partial x^2} + \lambda\frac{\partial^2\varepsilon_{22}}{\partial x^2} + 2\mu\frac{\partial^2\varepsilon_{12}}{\partial x\partial y} - \gamma\frac{\partial^2 T}{\partial x^2} = \rho\frac{\partial^2\varepsilon_{11}}{\partial t^2},\\
(\lambda+2\mu)\frac{\partial^2\varepsilon_{22}}{\partial y^2} + \lambda\frac{\partial^2\varepsilon_{11}}{\partial y^2} + 2\mu\frac{\partial^2\varepsilon_{12}}{\partial x\partial y} - \gamma\frac{\partial^2 T}{\partial y^2} = \rho\frac{\partial^2\varepsilon_{22}}{\partial t^2},\\
\mu(\frac{\partial^2\varepsilon_{12}}{\partial x^2} + \frac{\partial^2\varepsilon_{12}}{\partial y^2}) + (\lambda+\mu)(\frac{\partial^2\varepsilon_{11}}{\partial x\partial y} + \frac{\partial^2\varepsilon_{22}}{\partial x\partial y}) - \gamma\frac{\partial^2 T}{\partial x\partial y} = \rho\frac{\partial^2\varepsilon_{12}}{\partial t^2},\\
\lambda_0(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) - C_\varepsilon\frac{\partial T}{\partial t} - \gamma T\frac{\partial(\varepsilon_{11}+\varepsilon_{22})}{\partial t} = 0,
\end{aligned} \tag{18}$$

Coupled problems A and B have the following initial and boundary conditions: initial conditions

$$\begin{aligned}
T|_{t=0} = \tilde{T}_i, \varepsilon_{11}|_{t=0} = \xi_i, \varepsilon_{22t=0} = \psi_i, \varepsilon_{12}|_{t=0} = \zeta_i,\\
\frac{\partial\varepsilon_{11}}{\partial t}|_{t=0} = \xi_i^1, \frac{\partial\varepsilon_{22}}{\partial t}|_{t=0} = \psi_i^1, \frac{\partial\varepsilon_{12}}{\partial t}|_{t=0} = \zeta_i^1;
\end{aligned} \tag{19}$$

boundary conditions

$$\begin{aligned}
T(x,y,t)|_{x=0} = T_1, T(x,y,t)|_{x=l_1} = T_2,\\
T(x,y,t)|_{y=0} = T_3, T(x,y,t)|_{y=l_2} = T_4
\end{aligned} \tag{20}$$

$$\begin{aligned}
\varepsilon_{11}|_{x=0} = 0, \varepsilon_{11}|_{x=l_1} = 0, \varepsilon_{11}|_{y=0} = 0, \varepsilon_{11}|_{y=l_2} = 0,\\
\varepsilon_{22}|_{x=0} = 0_1, \varepsilon_{22}|_{x=l_1} = 0, \varepsilon_{22}|_{y=0} = 0\varepsilon_{22}|_{y=l_2} = 0,\\
\varepsilon_{12}|_{x=0} = 0, \varepsilon_{12}|_{x=l_1} = 0, \varepsilon_{12}|_{y=0} = 0, \varepsilon_{12}|_{x=l_2} = 0.
\end{aligned} \tag{21}$$

Note that the boundary conditions (21) are valid under the assumption that in the initial state the stresses and strains are equal to zero.

Note that the coupled thermoelasticity problem (1-6), in plane strain case, can be written with respect to displacements **(problem C)**

$$\begin{gathered}(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}+\mu\frac{\partial^2 u}{\partial y^2}+(\lambda+\mu)\frac{\partial^2 v}{\partial x\partial y}-(3\lambda+2\mu)\alpha\frac{\partial T}{\partial x}=\rho\frac{\partial^2 u}{\partial t^2},\\ (\lambda+2\mu)\frac{\partial^2 v}{\partial y^2}+\mu\frac{\partial^2 v}{\partial x^2}+(\lambda+\mu)\frac{\partial^2 u}{\partial x\partial y}-(3\lambda+2\mu)\alpha\frac{\partial T}{\partial y}=\rho\frac{\partial^2 v}{\partial t^2},\\ \lambda_0\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)-c_\varepsilon\frac{\partial T}{\partial t}-(3\lambda+2\mu)\alpha T\left(\frac{\partial^2 u}{\partial x\partial t}+\frac{\partial^2 v}{\partial y\partial t}\right)=0\end{gathered}\tag{22}$$

with corresponding initial

$$u|_{t=0}=\varphi_1,\ \left.\frac{\partial u}{\partial t}\right|_{t=0}=\psi_1,\ v|_{t=0}=\varphi_2,\ \left.\frac{\partial v}{\partial t}\right|_{t=0}=\psi_2,\ T|_{t=0}=T_0\tag{23}$$

and boundary conditions

$$\begin{gathered}u|_{x=0}=u_0,\quad u|_{x=\ell_1}=\bar u_0,\ u|_{y=0}={u'}_0,\quad u|_{y=\ell_2}=\bar u_0',\\ v|_{x=0}=v_0,\quad v|_{x=\ell_1}=\bar v_0,\ v|_{y=0}={v'}_0,\quad v|_{y=\ell_2}=\bar v_0',\\ T|_{x=0}=T_1(t),\quad T|_{x=\ell_1}=T_2(t),\quad T|_{y=0}={T_1}'(t),\quad T|_{y=\ell_2}={T'}_2(t)\ .\end{gathered}\tag{24}$$

4 Finite-difference equations for plane thermoelastic problems in strains

Considering nodal points $x_i=ih_1, y_j=jh_2(i,j=\overline{0,n}), t=k\tau\ (k=0,1,2,...)$ with meshes $h_1=l_1/N_1,\ \ h_2=l_2/N_2$ in the given domain in $t\geqslant 0,\quad 0\leqslant x\leqslant l$ and replacing the derivatives in equations (14) for problem B with finite-difference relations [4, 6], we can find the following grid equations

$$\begin{gathered}(\lambda+2\mu)\frac{\varepsilon^{11}_{i+1,j,k}-2\varepsilon^{11}_{i,j,k}+\varepsilon^{11}_{i-1,j,k}}{h_1^2}+\\ +\lambda\frac{\varepsilon^{22}_{i+1,j,k}-2\varepsilon^{22}_{i,j,k}+\varepsilon^{22}_{i-1,j,k}}{h_1^2}-\gamma\frac{T_{i+1,j,k}-2T_{i,j,k}+T_{i-1,j,k}}{h_1^2}\\ +2\mu\frac{\varepsilon^{12}_{i+1,j+1,k}-\varepsilon^{12}_{i+1,j-1,k}-\varepsilon^{12}_{i-1,j+1,k}+\varepsilon^{12}_{i-1,j-1,k}}{4h_1h_2}=\\ =\rho\frac{\varepsilon^{11}_{i,j,k+1}-2\varepsilon^{11}_{i,j,k}+\varepsilon^{11}_{i,j,k-1}}{\tau^2},\end{gathered}\tag{25}$$

$$\begin{gathered}(\lambda+2\mu)\frac{\varepsilon^{22}_{i,j+1,k}-2\varepsilon^{22}_{i,j,k}+\varepsilon^{22}_{i,j-1,k}}{h_2^2}+\\ +\lambda\frac{\varepsilon^{11}_{i,j+1,k}-2\varepsilon^{11}_{i,j,k}+\varepsilon^{11}_{i,j-1,k}}{h_2^2}-\gamma\frac{T_{i,j+1,k}-2T_{i,j,k}+2T_{i,j-1,k}}{h_2^2}\\ +2\mu\frac{\varepsilon^{12}_{i+1,j+1,k}-\varepsilon^{12}_{i+1,j-1,k}-\varepsilon^{12}_{i-1,j+1,k}+\varepsilon^{12}_{i-1,j-1,k}}{4h_1h_2}=\\ =\rho\frac{\varepsilon^{22}_{i,j,k+1}-2\varepsilon^{22}_{ijk}+\varepsilon^{22}_{i,j,k-1}}{\tau^2},\end{gathered}\tag{26}$$

$$\mu(\frac{\varepsilon^{12}_{i+1,j,k}-2\varepsilon^{12}_{i,j,k}+\varepsilon^{12}_{i-1,j,k}}{h_1^2}+\frac{\varepsilon^{12}_{i,j+1,k}-2\varepsilon^{12}_{i,j,k}+\varepsilon^{12}_{i,j-1,k}}{h_2^2})-$$
$$-\frac{T_{i+1,j+1,k}-T_{i+1,j-1,k}-T_{i-1,j+1,k}+T_{i-1,j-1,k}}{4h_1h_2}$$
$$+(\lambda+\mu)(\frac{\varepsilon^{11}_{i+1,j+1,k}-\varepsilon^{11}_{i+1,j-1,k}-\varepsilon^{11}_{i-1,j+1,k}+\varepsilon^{11}_{i-1,j-1,k}}{4h_1h_2}+ \tag{27}$$
$$+\frac{\varepsilon^{22}_{i+1,j+1,k}-\varepsilon^{22}_{i+1,j-1,k}-\varepsilon^{22}_{i-1,j+1,k}+\varepsilon^{22}_{i-1,j-1,k}}{4h_1h_2})=$$
$$=\rho\frac{\varepsilon^{12}_{i,j,k+1}-2\varepsilon^{12}_{i,j,k}+\varepsilon^{12}_{i,j,k-1}}{\tau^2},$$

$$\lambda_0(\frac{T_{i+1,j,k}-2T_{i,j,k}+T_{i-1,j,k}}{h_1^2}+\frac{T_{i,j+1,k}-2T_{i,j,k}+T_{i,j-1,k}}{h_2^2})-C_\varepsilon\frac{2T_{i,j,k+1}-2T_{i,j,k-1}}{2\tau}-$$
$$-\gamma 2T^0_{i,j,k}(\frac{\varepsilon^{11}_{i,j,k+1}-\varepsilon^{11}_{i,j,k-1}}{2\tau}+\frac{\varepsilon^{22}_{i,j,k+1}-\varepsilon^{22}_{i,j,k-1}}{2\tau})=0. \tag{28}$$

These equations within the domain have a second order of approximation $O(h^2,\tau^2)$ and are explicit. Therefore, by solving these difference equations (25-28) for $\varepsilon_{i,k+1}$ and $T_{i,k+1}$ respectively, we obtain the following recurrence relations i.e.

$$\varepsilon^{11}_{i,j,k+1}=\frac{\tau^2}{\rho}((\lambda+2\mu)\frac{\varepsilon^{11}_{i+1,j,k}-2\varepsilon^{11}_{i,j,k}+\varepsilon^{11}_{i-1,j,k}}{h_1^2}+$$
$$+\lambda\frac{\varepsilon^{22}_{i+1,j,k}-2\varepsilon^{22}_{i,j,k}+\varepsilon^{22}_{i-1,j,k}}{h_1^2}-\gamma\frac{T_{i+1,j,k}-2T_{i,j,k}+T_{i-1,j,k}}{h_1^2}+ \tag{29}$$
$$+2\mu\frac{\varepsilon^{12}_{i+1,j+1,k}-\varepsilon^{12}_{i+1,j-1,k}-\varepsilon^{12}_{i-1,j+1,k}+\varepsilon^{12}_{i-1,j-1,k}}{4h_1h_2})+2\varepsilon^{11}_{i,j,k}-\varepsilon^{11}_{i,j,k-1},$$

$$\varepsilon^{22}_{i,j,k+1}=\frac{\tau^2}{\rho}((\lambda+2\mu)\frac{\varepsilon^{22}_{i,j+1,k}-\varepsilon^{22}_{i,j,k}+\varepsilon^{22}_{i,j-1,k}}{h_2^2}+$$
$$+\lambda\frac{\varepsilon^{11}_{i,j+1,k}-\varepsilon^{11}_{i,j,k}+\varepsilon^{11}_{i,j-1,k}}{h_2^2}-\gamma\frac{T_{i,j+1,k}-2T_{i,j,k}+T_{i,j-1,k}}{h_2^2}+ \tag{30}$$
$$+2\mu\frac{\varepsilon^{12}_{i+1,j+1,k}-\varepsilon^{12}_{i+1,j-1,k}-\varepsilon^{12}_{i-1,j+1,k}+\varepsilon^{12}_{i-1,j-1,k}}{4h_1h_2})+2\varepsilon^{22}_{i,j,k}-\varepsilon^{22}_{i,j,k-1},$$

$$\varepsilon^{12}_{i,j,k+1}=\frac{\tau^2}{\rho}(\mu(\frac{\varepsilon^{12}_{i+1,j,k}-2\varepsilon^{12}_{i,j,k}+\varepsilon^{12}_{i-1,j,k}}{h_1^2}+\frac{\varepsilon^{12}_{i,j+1,k}-\varepsilon^{12}_{i,j,k}+\varepsilon^{12}_{i,j-1,k}}{h_2^2})-$$
$$-\frac{T_{i+1,j+1,k}-T_{i+1,j-1,k}-T_{i-1,j+1,k}+T_{i-1,j-1,k}}{4h_1h_2}+$$
$$+(\lambda+\mu)(\frac{\varepsilon^{11}_{i+1,j+1,k}-\varepsilon^{11}_{i+1,j-1,k}-\varepsilon^{11}_{i-1,j+1,k}+\varepsilon^{11}_{i-1,j-1,k}}{4h_1h_2} \tag{31}$$
$$+\frac{\varepsilon^{22}_{i+1,j+1,k}-\varepsilon^{22}_{i+1,j-1,k}-\varepsilon^{22}_{i-1,j+1,k}+\varepsilon^{22}_{i-1,j-1,k}}{4h_1h_2}))+2\varepsilon^{12}_{i,j,k}-\varepsilon^{12}_{i,j,k-1},$$

$$2T_{i,j,k+1}=\frac{2\tau}{C_\varepsilon}(\lambda_0(\frac{T_{i+1,j,k}-2T_{i,j,k}+T_{i-1,j,k}}{h_1^2}+\frac{T_{i,j+1,k}-2T_{i,j,k}+T_{i,j-1,k}}{h_2^2})-$$
$$-\gamma 2T^0_{i,j,k}(\frac{\varepsilon^{11}_{i,j,k+1}-\varepsilon^{11}_{i,j,k-1}}{2\tau}+\frac{\varepsilon^{22}_{i,j,k+1}-\varepsilon^{22}_{i,j,k-1}}{2\tau}))+T_{i,j,k-1}. \tag{32}$$

Using relations (29-32) may be find the values of $\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12}$ and T at t_{k+1}, if the values of these functions are known at $k=0$ and $k=1$.

It is known that the time step τ is very small compared to h for explicit schemes i.e. $\frac{\tau^2}{h} < 1$ [6]. It is possible to construct difference schemes for which there are no restrictive conditions on the grid steps in x and t. Why do we replace the index k in the first terms of equations (25) with $k+1$, then the difference scheme becomes implicit

$$
\begin{gathered}
(\lambda+2\mu)\frac{\varepsilon^{11}_{i+1,j,k+1}-2\varepsilon^{11}_{i,j,k+1}+\varepsilon^{11}_{i-1,j,k+1}}{h_1^2}+\\
+\lambda\frac{\varepsilon^{22}_{i+1,j,k}-2\varepsilon^{22}_{i,j,k}+\varepsilon^{22}_{i-1,j,k}}{h_1^2}-\gamma\frac{T_{i+1,j,k}-2T_{i,j,k}+T_{i-1,j,k}}{h_1^2}+\\
+2\mu\frac{\varepsilon^{12}_{i+1,j+1,k}-\varepsilon^{12}_{i+1,j-1,k}-\varepsilon^{12}_{i-1,j+1,k}+\varepsilon^{12}_{i-1,j-1,k}}{4h_1h_2}=\\
=\rho\frac{\varepsilon^{11}_{i,j,k+1}-2\varepsilon^{11}_{i,j,k}+\varepsilon^{11}_{i,j,k-1}}{\tau^2}
\end{gathered}
\tag{33}
$$

and may be written in the following tridiagonal form

$$
a\varepsilon^{11}_{i+1,j,k+1}+b\varepsilon^{11}_{i,j,k+1}+c\varepsilon^{11}_{i-1,j,k+1}=f^x_{i,j}, \tag{34}
$$

where $a, b, c, f^x_{i,j}-$ coefficients.

Equations (26) and (27), similar to (28), can be written in the following form, i.e.

$$
\begin{aligned}
a'_i\varepsilon^{22}_{i,j+1,k+1}&+b'_i\varepsilon^{22}_{i,j,k+1}+c'_i\varepsilon^{22}_{i,j-1,k+1}=f^{,}_{ijk}\\
\dot a_i\varepsilon^{12}_{i+1,j,k+1}&+\dot b_i\varepsilon^{12}_{i,j,k+1}+\dot c_i\varepsilon^{12}_{i-1,j,k+1}=f^{xx}_{ijk},\\
\tilde a_i\varepsilon^{12}_{i,j+1,k+1}&+\tilde b_i\varepsilon^{12}_{i,j,k+1}+\tilde c_i\varepsilon^{12}_{i,j-1,k+1}=f^{y}_{ijk},\\
AT_{i+1,j,k+1}&+BT_{i,j,k+1}+CT_{i-1,j,k+1}=F^{xx}_{ijk},\\
\tilde AT_{i,j+1,k+1}&+\tilde BT^{12}_{i,j,k+1}+\tilde CT^{12}_{i,j-1,k+1}=\tilde F^{y}_{ijk}.
\end{aligned}
\tag{35}
$$

Equations (25) and (26) may be solved using the variable direction method.

In a similar way, can be found finite-difference equations for problem A and solved using the variable direction method.

Finite-difference equations for problem C (22)-(24) have the following form

$$
\left.
\begin{array}{l}
(\lambda+2\mu)\frac{u_{i+1,j,k}-2u_{i,j,k}+u_{i-1,j,k}}{h_1^2}+(\lambda+\mu)\frac{v_{i+1,j+1,k}-v_{i-1,j+1,k}-v_{i+1,j-1,k}+v_{i-1,j-1,k}}{4h_1h_2}+\\
\mu\frac{u_{i,j+1,k}-2u_{i,j,k}+u_{i,j-1,k}}{h_2^2}-\gamma\frac{T_{i+1,j,k}-T_{i-1,j,k}}{2h_1}=\rho\frac{u_{i,j,k+1}-2u_{i,j,k}+u_{i,j,k-1}}{\tau^2},\\
(\lambda+2\mu)\frac{v_{i,j+1,k}-2v_{i,j,k}+v_{i,j-1,k}}{h_2^2}+(\lambda+\mu)\frac{u_{i+1,j+1,k}-u_{i-1,j+1,k}-u_{i+1,j-1,k}+u_{i-1,j-1,k}}{4h_1h_2}+\\
\mu\frac{v_{i+1,j,k}-2v_{i,j,k}+v_{i-1,j,k}}{h_1^2}-\gamma\frac{T_{i,j+1,k}-T_{i,j-1,k}}{2h_2}=\rho\frac{v_{i,j,k+1}-2v_{i,j,k}+v_{i,j,k-1}}{\tau^2},
\end{array}
\right\}
\tag{36}
$$

$$
\begin{gathered}
\lambda_0\left(\frac{T_{i+1,j,k}-2T_{i,j,k}+T_{i-1,j,k}}{h_1^2}+\frac{2T_{i,j+1,k}-2T_{i,j,k}+2T_{i,j-1,k}}{h_2^2}\right)-c_\varepsilon\frac{2T_{i,j,k+1}-2T_{i,j,k}}{\tau}-\\
-\gamma 2T_{i,j,k}\left(\frac{u_{i+1,j,k+1}-u_{i-1,j,k+1}-u_{i+1,j,k-1}+u_{i-1,j,k-1}}{4h_1\tau}+\right.\\
\left.+\frac{v_{i+1,j,k+1}-v_{i-1,j,k+1}-v_{i+1,j,k-1}+v_{i-1,j,k-1}}{4h_2\tau}\right)=0
\end{gathered}
\tag{37}
$$

and having resolved the resulting difference equations for $u_{i,j,k+1}$, $v_{i,j,k+1}$, $T_{i,j,k+1}$, we respectively obtain

$$\begin{gathered} u_{i,j,k+1} = \frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{u_{i+1,j,k}-2u_{i,j,k}+u_{i-1,j,k}}{h_1^2} + \mu\frac{u_{i,j+1,k}-2u_{i,j,k}+u_{i,j-1,k}}{h_2^2} + \right. \\ \left. +(\lambda+\mu)\frac{v_{i+1,j+1,k}-v_{i-1,j+1,k}-v_{i+1,j-1,k}+v_{i-1,j-1,k}}{4h_1h_2} - \gamma\,\frac{T_{i+1,j,k}-T_{i-1,j,k}}{2h_1}\right) + \\ +2u_{i,j,k}-u_{i,j,k-1}, \end{gathered} \tag{38}$$

$$\begin{gathered} v_{i,j,k+1} = \frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{v_{i,j+1,k}-2v_{i,j,k}+v_{i,j-1,k}}{h_2^2} + \mu\frac{v_{i+1,j,k}-2v_{i,j,k}+v_{i-1,j,k}}{h_1^2} + \right. \\ \left. +(\lambda+\mu)\frac{u_{i+1,j+1,k}-u_{i-1,j+1,k}-u_{i+1,j-1,k}+u_{i-1,j-1,k}}{4h_1h_2} - \gamma\,\frac{T_{i,j+1,k}-T_{i,j-1,k}}{2h_2}\right) + \\ +2v_{i,j,k}-v_{i,j,k-1}, \end{gathered} \tag{39}$$

$$\begin{gathered} T_{i,j,k+1} = \frac{\tau}{c_\varepsilon}\left(\lambda_0(\frac{2T_{i+1,j,k}-2T_{i,j,k}+T_{i-1,j,k}}{h_1^2} + \frac{T_{i,j+1,k}-2T_{i,j,k}+T_{i,j-1,k}}{h_2^2}) - \right. \\ -\gamma T_{ij}^k\left(\frac{u_{i+1,j,k+1}-u_{i-1,j,k+1}-u_{i+1,j,k-1}+u_{i-1,j,k-1}}{4h_1\tau} + \right. \\ \left.\left. + \frac{v_{i+1,j,k+1}-v_{i-1,j,k+1}-v_{i+1,j,k-1}+v_{i-1,j,k-1}}{4h_2\tau}\right)\right) + T_{i,j,k}. \end{gathered} \tag{40}$$

Using equations (38)-(40), you can find the values of u, v, T on layers t_{k+1}, based on the known values of functions on the two previous layers ($k=0 \quad and \quad k=1$) according to initial and boundary conditions.

$$\begin{gathered} u_{i,j,1} = \frac{1}{2}\left(\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{u_{i+1,j,0}-2u_{i,j,0}+u_{i-1,j,0}}{h_1^2} + \mu\frac{u_{i,j+1,0}-2u_{i,j,0}+u_{i,j-1,0}}{h_2^2} + \right.\right. \\ \left. +(\lambda+\mu)\frac{v_{i+1,j+1,0}-v_{i-1,j+1,0}-v_{i+1,j-1,0}+v_{i-1,j-1,0}}{4h_1h_2} - \gamma\,\frac{T_{i+1,j,0}-T_{i-1,j,0}}{2h_1}\right) + \\ \left. +2u_{i,j,0} + 2\tau\psi_1\right), \end{gathered} \tag{41}$$

$$\begin{gathered} v_{i,j,1} = \frac{1}{2}\left(\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{v_{i,j+1,0}-2v_{i,j,0}+u_{i,j-1,0}}{h_2^2} + \mu\frac{v_{i+1,j,0}-2v_{i,j,0}+v_{i-1,j,0}}{h_1^2} + \right.\right. \\ \left. +(\lambda+\mu)\frac{u_{i+1,j+1,0}-u_{i-1,j+1,0}-u_{i+1,j-1,0}+u_{i-1,j-1,0}}{4h_1h_2} - \gamma\,\frac{T_{i,j+1,0}-T_{i,j-1,0}}{2h_2}\right) + \\ \left. +2v_{i,j,0} + 2\tau\psi_2\right), \end{gathered} \tag{42}$$

$$\begin{gathered} T_{i,j,1} = \frac{\tau}{c_\varepsilon}\left(\lambda_0\left(\frac{T_{i+1,j,0}-2T_{i,j,0}+T_{i-1,j,0}}{h_1^2} + \frac{T_{i,j+1,0}-2T_{i,j,0}+T_{i,j-1,0}}{h_2^2}\right) - \right. \\ -\gamma T_{ij}^0\left(\frac{u_{i+1,j,1}-u_{i-1,j,1}-u_{i+1,j,0}+u_{i-1,j,0}}{2h_1\tau} + \right. \\ \left.\left. + \frac{v_{i,j+1,1}-v_{i,j-1,1}-v_{i,j+1,0}+v_{i,j-1,0}}{2h_2\tau}\right)\right) + T_{i,j,0}. \end{gathered} \tag{43}$$

5 Numerical results

Explicit and implicit schemes of the coupled thermoelasticity problem in strains (17-21) were solved by recurrence formulas and the variable direction method, under the following initial and boundary conditions:

$$T|_{t=0} = T_0 + T_0 \sin(\frac{\pi x_i}{l_1}) \sin(\frac{\pi y_j}{l_2}),$$
$$\varepsilon_{11}|_{t=0} = 0, \varepsilon_{22}|_{t=0} = 0, \ \ \varepsilon_{12}|_{t=0} = 0, \quad (44)$$
$$\frac{\partial \varepsilon_{11}}{\partial t}|_{t=0} = 0, \frac{\partial \varepsilon_{22}}{\partial t}|_{t=0} = 0, \frac{\partial \varepsilon_{12}}{\partial t}|_{t=0} = 0;$$

$$\begin{aligned} &\varepsilon_{11}|_{x=0} = 0, \varepsilon_{11}|_{x=l_1} = 0, \varepsilon_{11}|_{y=0} = 0, \varepsilon_{11}|_{y=l_2} = 0, \\ &\varepsilon_{22}|_{x=0} = 0, \varepsilon_{22}|_{x=l_1} = 0, \varepsilon_{22}|_{y=0} = 0, \varepsilon_{22}|_{y=l_2} = 0, \\ &\varepsilon_{12}|_{x=0} = 0, \varepsilon_{12}|_{x=l_1} = 0, \varepsilon_{12}|_{y=0} = 0, \varepsilon_{12}|_{y=l_2} = 0, \\ &T|_{x=0} = T_0, T|_{x=l_1} = T_0, T|_{y=0} = T_0, T|_{y=l_2} = T_0. \end{aligned} \quad (45)$$

Regarding displacements has the following form:

$$\begin{aligned} &\varepsilon_{11}|_{x=0} = 0, \varepsilon_{11}|_{x=l_1} = 0, \varepsilon_{11}|_{y=0} = 0, \varepsilon_{11}|_{y=l_2} = 0, \\ &\varepsilon_{22}|_{x=0} = 0, \varepsilon_{22}|_{x=l_1} = 0, \varepsilon_{22}|_{y=0} = 0, \varepsilon_{22}|_{y=l_2} = 0, \\ &\varepsilon_{12}|_{x=0} = 0, \varepsilon_{12}|_{x=l_1} = 0, \varepsilon_{12}|_{y=0} = 0, \varepsilon_{12}|_{y=l_2} = 0, \\ &T|_{x=0} = T_0, T|_{x=l_1} = T_0, T|_{y=0} = T_0, T|_{y=l_2} = T_0; \end{aligned} \quad (46)$$

$$\begin{aligned} u_{0,j,k} &= 0, \quad u_{N_1,j,k} = 0, \quad u_{i,0,k} = 0, \quad u_{i,N_2,k} = 0, \\ v_{0,j,k} &= 0, \quad v_{N_1,j,k} = 0, \quad v_{i,0,k} = 0, \quad v_{i,N_2,k} = 0, \\ T_{0,j,k} &= T_0, \quad T_{N_1,j,k} = T_0, \quad T_{i,0,k} = T_0, \quad T_{i,N_2,k} = T_0 \end{aligned} \quad (47)$$

with a initial data

$$T_0 = 15, \lambda = 0.78, \lambda_0 = 0.06, \alpha = 0.05, \mu = 0.5,$$
$$\rho = 0.86, c_\varepsilon = 3.4, h_1 = h_2 = 0.1, l_1 = l_2 = 1.$$

Using the tables, you can compare the values for strains obtained on explicit and implicit schemes for problems A and B. A similar comparison was made for temperature.

Table 1 Values of ε_{11} at $t = 0.05$ (explicit scheme) problem A

	x=0	*x=0.1*	*x=0.2*	*x=0.3*	*x=0.4*	*x=0.5*
y=0	0.000	0.000	0.000	0.000	0.000	0.000
y=0.1	0.000	0.003	0.006	0.009	0.010	0.011
y=0.2	0.000	0.006	0.012	0.017	0.020	0.021
y=0.3	0.000	0.009	0.017	0.023	0.027	0.028
y=0.4	0.000	0.010	0.020	0.027	0.032	0.033
y=0.5	0.000	0.011	0.021	0.028	0.033	0.035

Table 2 Values of ε_{11} at $t = 0.05$ (explicit scheme) problem B

	x=0	*x=0.1*	*x=0.2*	*x=0.3*	*x=0.4*	*x=0.5*
y=0	0.000	0.000	0.000	0.000	0.000	0.000
y=0.1	0.000	0.002	0.003	0.005	0.007	0.007
y=0.2	0.000	0.003	0.008	0.013	0.016	0.017
y=0.3	0.000	0.005	0.013	0.019	0.023	0.025
y=0.4	0.000	0.007	0.016	0.023	0.028	0.030
y=0.5	0.000	0.007	0.017	0.025	0.030	0.032

Table 3 Values of ε_{11} at $t = 0.05$ (implicit scheme) problem A

	x=0	*x=0.1*	*x=0.2*	*x=0.3*	*x=0.4*	*x=0.5*
y=0	0.000	0.000	0.000	0.000	0.000	0.000
y=0.1	0.000	0.003	0.006	0.009	0.010	0.011
y=0.2	0.000	0.006	0.012	0.017	0.020	0.021
y=0.3	0.000	0.009	0.017	0.023	0.027	0.028
y=0.4	0.000	0.010	0.020	0.027	0.032	0.033
y=0.5	0.000	0.011	0.021	0.028	0.033	0.035

Table 4 Values of ε_{11} at $t = 0.05$ (implicit scheme) problem B

	x=0	*x=0.1*	*x=0.2*	*x=0.3*	*x=0.4*	*x=0.5*
y=0	0.000	0.000	0.000	0.000	0.000	0.000
y=0.1	0.000	0.002	0.003	0.005	0.007	0.007
y=0.2	0.000	0.003	0.008	0.013	0.016	0.017
y=0.3	0.000	0.005	0.013	0.019	0.023	0.024
y=0.4	0.000	0.007	0.016	0.023	0.028	0.030
y=0.5	0.000	0.007	0.017	0.024	0.030	0.032

Table 5 Values of T at $t = 0.05$ (explicit scheme) problem A

	x=0	*x=0.1*	*x=0.2*	*x=0.3*	*x=0.4*	*x=0.5*
y=0	15.000	15.000	15.000	15.000	15.000	15.000
y=0.1	15.000	16.409	17.680	18.688	19.335	19.558
y=0.2	15.000	17.680	20.096	22.012	23.241	23.665
y=0.3	15.000	18.688	22.012	24.647	26.338	26.920
y=0.4	15.000	19.335	23.241	26.338	28.325	29.009
y=0.5	15.000	19.558	23.665	26.920	29.009	29.728

Table 6 Values of T at $t = 0.05$ (explicit scheme) problem B

	x=0	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5
y=0	15.000	15.000	15.000	15.000	15.000	15.000
y=0.1	15.000	16.410	17.682	18.691	19.338	19.561
y=0.2	15.000	17.682	20.099	22.015	23.245	23.669
y=0.3	15.000	18.691	22.015	24.651	26.342	26.925
y=0.4	15.000	19.338	23.245	26.342	28.329	29.013
y=0.5	15.000	19.561	23.669	26.925	29.013	29.732

Table 7 Values of ε_{11} at $t = 0.05$ (explicit scheme) problem C

	x=0	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5
y=0	0.000	0.000	0.000	0.000	0.000	0.000
y=0.1	0.000	0.002	0.006	0.009	0.010	0.011
y=0.2	0.000	0.003	0.011	0.016	0.019	0.020
y=0.3	0.000	0.005	0.014	0.022	0.026	0.028
y=0.4	0.000	0.005	0.017	0.026	0.031	0.033
y=0.5	0.000	0.006	0.018	0.028	0.033	0.034

Table 8 Values of T at $t = 0.05$ (explicit scheme) problem C

	x=0	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5
y=0	15.000	15.000	15.000	15.000	15.000	15.000
y=0.1	15.000	16.426	17.692	18.706	19.357	19.581
y=0.2	15.000	17.692	20.076	21.983	23.207	23.629
y=0.3	15.000	18.706	21.983	24.604	26.286	26.866
y=0.4	15.000	19.357	23.207	26.286	28.262	28.943
y=0.5	15.000	19.581	23.629	26.866	28.943	29.659

Figure 1 shows the distribution of deformations by coordinate and time obtained using an explicit scheme for problem B.

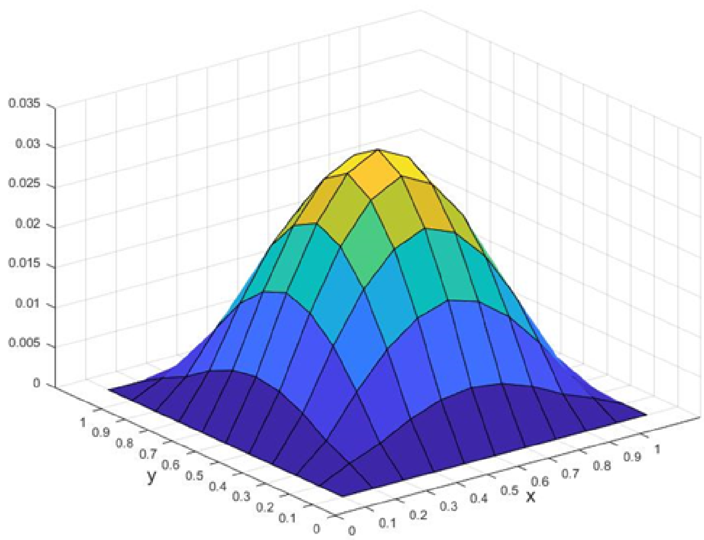

Figure 1 Distribution graph of the strain tensor (explicit scheme) at $t = 0.05$ (problem B)

In Fig. 2 compares the curves showing the change in deformation over time at the midpoint of the rectangle, constructed from the results of the obtained recurrent formulas (explicit scheme) and the variable direction method (implicit scheme) for problem B.

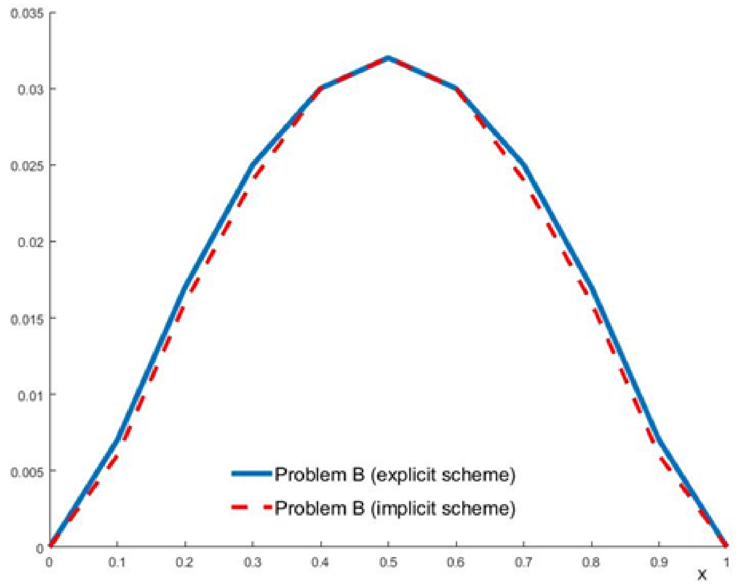

Figure 2 Comparison of the results of the deformation tensor over time at $t = 0.05, y = 0.5$ (problem B)

Figure 3 compares the results of task A and B in time. Figure 4 compares the results of the deformation tensor obtained on the basis of problem A with an explicit and implicit scheme. Figure 5 compares the numerical results based on Problems A, B, and C.

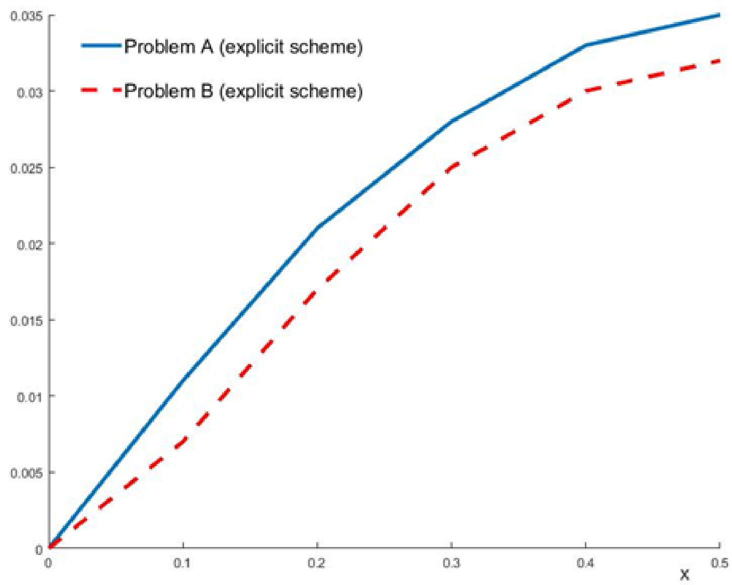

Figure 3 Comparison of the strain tensor distribution over time $t = 0.05, y = 0.5$ based on problems A and B

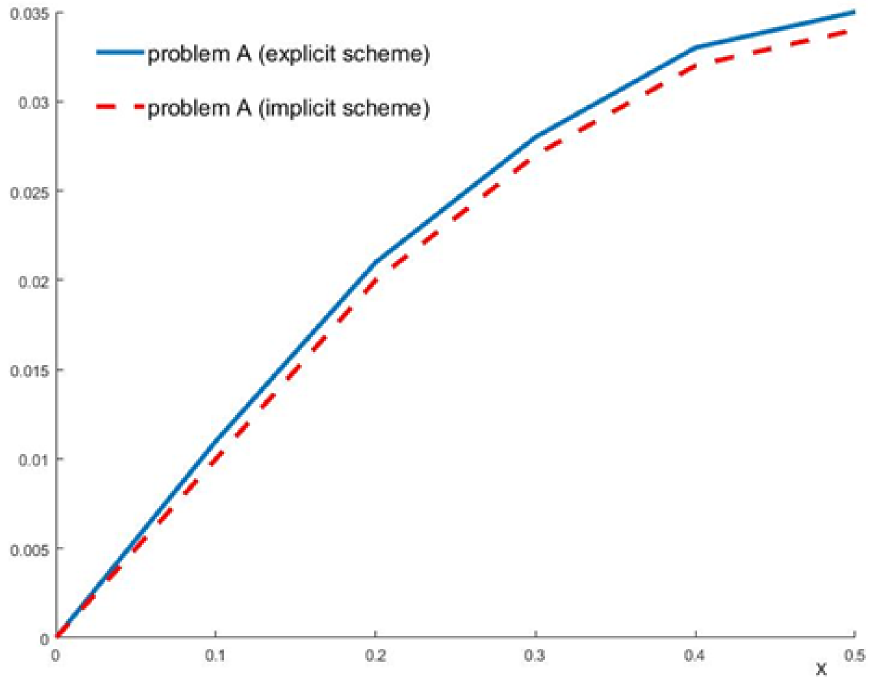

Figure 4 Comparisons of the distribution of the strain tensor over time $t = 0.05, y = 0.5$ based on problem A obtained with the explicit and implicit scheme

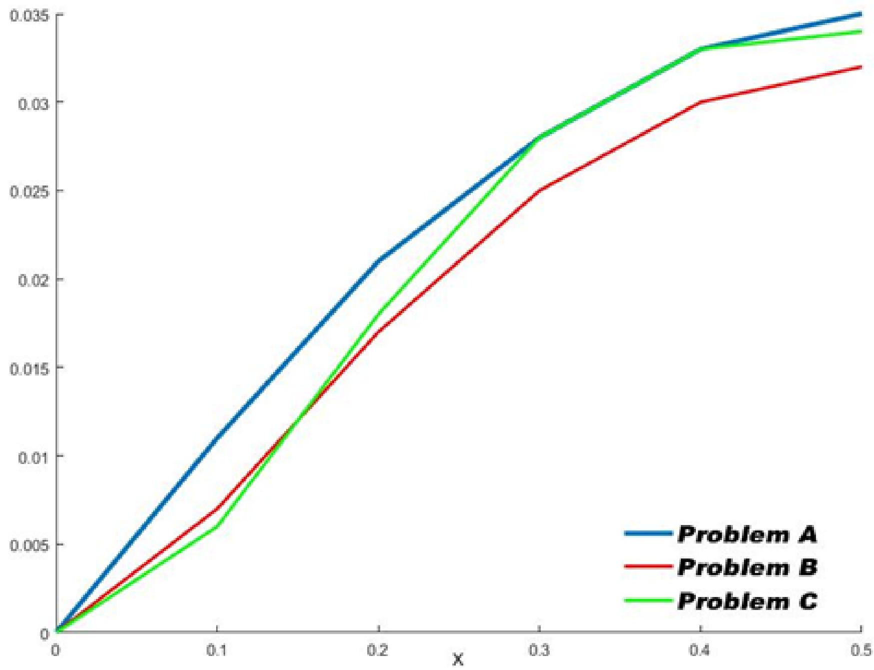

Figure 5 Comparisons of the distribution of the strain tensor over time $t = 0.05, y = 0.5$ based on problems A, B, and C obtained with an explicit scheme

A comparison of the results from the tables and figures shows that the numerical results found by recurrence relations and by the variable direction method are quite close, which ensures the validity of the formulated boundary value problems and the reliability of the obtained numerical results.

6 Conclusion

• Based on motion equation and the Duhamel-Neumann relation, Saint-Venant compatibility conditions are written as a system of six differential equations in strains and temperature;

• Differential equations of thermoelasticity together with the heat influx equation and corresponding initial and boundary conditions constitute coupled thermoelasticity problem in strains (Problem A);

• Considering instead of the first three of the six differential equations in strains, three motion equations we can to formulate an equivalent coupled problem in strains (problem B);

• Along with the usual kinematic and natural boundary conditions, initial and boundary conditions can be set relative to the strain tensor;

• Implicit and explicit finite-difference equations of related thermoelasticity problems can be solved by sequential application of the method of sweeping along the corresponding axes and recurrent relations between strains and temperature;

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НОВЫЕ СВЯЗАННЫЕ КРАЕВЫЕ ЗАДАЧИ ТЕРМОУПРУГОСТИ В ДЕФОРМАЦИЯХ

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Настоящая работа посвящена формулировке и численному решению связанных краевых задач термоупругости относительно деформаций. Предложены две эквивалентные связанные краевые задачи термоупругости относительно деформаций и температуры. Первая состоит из шести дифференциальных уравнений термоупругости найденных в рамках условий совместности деформаций Сен-Венана и уравнения притока тепла с соответствующими начальными и краевыми условиями. Во втором случае, первые три из шести дифференциальных уравнений термоупругости заменена с тремя продифференцированными уравнения движения. Справедливость сформулированных двух краевых задач термоупругости обоснованы сравнением их численных, полученных по методу прогонки и рекуррентных соотношений, а также решением аналогичной связанной задачи относительно перемещений.

Ключевые слова: условие совместности Сен-Венана, метод конечных разностей, явные и неявные схемы, метод переменного направления.

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