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MULTIDIMENSIONAL MATHEMATICAL MODEL OF SIMULTANEOUS HEAT AND MOISTURE TRANSFER DURING DRYING AND STORAGE OF RAW COTTON IN OPEN AREAS

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A multidimensional mathematical model of simultaneous heat and moisture transfer processes in inhomogeneous porous bodies is proposed, considering internal heat and moisture release, heat and moisture exchange with the environment. Based on the usage of an implicit finite-difference scheme with the second order of precision in time and space variables, an effective numerical solution for resolving issues has been created. Based on the developed numerical algorithm the software was created for studying and analysing the processes of heat and moisture transfer during the storage and drying of raw cotton in open areas which makes it possible to identify and forecast changes in temperature and humidity at arbitrary points of cotton.

Keywords: mathematical model, finite-difference scheme, heat transfer, moisture transfer, raw cotton.

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1 Introduction

Scientific research in mathematical modelling, numerical methods, and the development of software and tools for solving issues with heat and moisture transport in porous materials is being successfully applied widely. In addition to producing more accurate predictions of changes in agricultural product indicators like internal temperature and humidity, these studies can provide qualitatively new information about the processes under investigation and lead to technological advancements in equipment and techniques to improve the conditions for agricultural product processing and storage in open areas under the influence of solar radiation.

The problems of developing the theoretical foundations and methodology for modelling complex processes of heat and mass transfer are devoted to the work of a number of prominent scientists, such as A.V. Lykov, Yu.A. Mikhailov [1], P.V. Akulich, N.N. Grinchik, [2], LI.YI and Zhu.Qingyong [3], D.A.Nield and A.Bejan [4] and others. Problems of mathematical modeling of the processes of simultaneous heat and moisture transfer, drying and storage of various materials are considered in the works of A.Afanasyev, B.Siplivy [5], N. Ravshanov [6], A.Mamatov, A.Parpiev and A.Kayumov [7], I.Toğrul,D.Pehlivan [8], T.J.Afolabi and S.E.Agarry [9], V.Patel et al., [10], N.Wang and J.G.Brennan [11], G.Arunsandeep, V.P.Chandramohan [12] et al. The Philippe and De Vries model [13], A.V.Lykov [14], and Whitaker [15] are now the most commonly studied models used to describe the processes of heat and moisture transfer in capillary-porous media. The ideas of Fourier's law of heat conduction, Fick's law of gas diffusion, Darcy's law of liquid diffusion, and mass and energy conservation were used to build these models. One feature of these models is the ability to include control potentials, including partial pressure, relative humidity, and water content in porous bodies. A set of partial differential equations presented by A.V.Lykov describes the processes of linked heat and moisture transfer inside a wet porous body during drying and has the following form [14]:

$$\frac{\partial u}{\partial \tau} = a_{11} \nabla^2 u + a_{12} \nabla^2 T; \tag{1}$$

$$\frac{\partial T}{\partial \tau} = a_{22} \nabla^2 T + a_{21} \nabla^2 u, \tag{2}$$

where coefficients $a_{11}, a_{12}, a_{22}, a_{21}$ are determined by the relations:

$a_{11} = a_m = \frac{\lambda_2}{\rho_2}$ – diffusion coefficient of moisture;

$a_{12} = a_m^{=} a_m \delta$ – coefficient of thermal diffusion of wet bodies;

$a_{22} = a + a_{m1}^T \frac{r_{12}}{} = \frac{\lambda_1}{\rho_1} + a_{m1}^T \frac{r_{12}}{}$ – coefficient of diffusion of heat;

$a_{21} = a_{m1} \frac{r_{12}}{}$ – coefficient of thermal diffusion of wet bodies;

r_{12} – capillary radius; δ – relative moisture thermal diffusion coefficient, usually experimentally determined by the formula $\delta = \frac{a_m^T}{a_m}$.

Jen Y. Liu's [16] proposed an analytical method for solving the A.V.Lykov heat and mass transfer equations with time-dependent boundary conditions. The solution consists of the sum of solutions of inhomogeneous equations. Only specific solutions change as the boundary conditions vary, while the homogenous solutions stay unchanged. Numerical findings on the drying of a porous material example demonstrate that drying times and heat absorption can be shortened until the required moisture content is attained by concurrently increasing temperature and decreasing the equilibrium mass transfer potential over time.

The paper [17] presents a two-dimensional model for the investigation of heat and moisture transmission across porous wood construction materials. A non-stationary coupled model for heat transfer and moisture exchange in low-temperature wood materials is presented by researchers. Next, using an implicit iterative method, the two non-linear partial differential equations arising from the coupled model are numerically solved. Numerical results of the prospective temperature and humidity change are compared with experimental values published in the scientific literature.

This [18] study offers a mathematical formulation of difficulties concerning the impact of a concentrated source on the surface of a half-space in both two- and three-dimensional formulations, as well as the influence of a volumetric heat source in an infinite space. A method is developed to address the issue of a consistent, concentrated heat source. Using the superposition principle and the influence function of the volumetric source, the solution is derived in integral form. A comparison is made with classical solutions in the case of a parabolic type heat equation. It is demonstrated that accounting for the finite speed of thermal waves only has a major impact on the heating process during the initial, comparatively brief phase of exposure to the heat source.

Cotton seed open sun drying has been studied experimentally and reported in the research work [19]. Using the hot air oven method, the initial moisture content of cotton seeds was assessed to be 14.65% wet-basis. The drying process reduced the moisture content from 14.65% to 6.37% wet basis in 20 hours. It happened during a declining rate phase without a steady rate period. In order to choose the best drying model for sun-drying cotton seeds, experimental moisture ratio findings were fitted with anticipated moisture ratio values.The work of the aforementioned scholars as well as many other researchers has made it possible to comprehensively analyze the processes of heat and moisture exchange that occur during the processing and storage of agricultural products. The development and improvement of mathematical models that take into consideration the effects of external and internal phenomena, such as solar radiation, ambient temperature and moisture, and self-heating, which have a major influence on the processes of heat and moisture exchange in cotton, has not yet received enough attention.

2 Problem statement

The most inclusive set of equations are (1) and (2), which apply to drying porous wet materials as well as all types of heat and moisture transfer. The object of the study is a bundle of raw cotton stored in an open space which has a shape close to a rectangular parallelepiped, the upper boundaries of which communicate with the environment, and the lower boundary is thermally insulated.

The following system of differential equations is proposed as a mathematical model of simultaneous heat and moisture transfer in the heterogeneous porous medium, which takes into account moisture and heat exchange with the environment, sources of heat and moisture release inside the body, and flow solar radiation:

$$\frac{\partial T}{\partial \tau} = div\left(a\,\nabla T\right) + div\left(\delta\,\nabla u\right) + f, \tag{3}$$

$$\frac{\partial u}{\partial \tau} = div\left(\delta\,\nabla u\right) + div\left(a\,\nabla T\right) + q \tag{4}$$

with initial

$$T\left(x, y, z, 0\right) = T_0\left(x, y, z\right);\;\; u\left(x, y, z, 0\right) = u_0\left(x, y, z\right) \tag{5}$$

and boundary conditions

$$\lambda_1 \frac{\partial T}{\partial x}\bigg|_{x=0} = -\beta_1\left(T_{oc} - T\left(0, y, z, \tau\right)\right) - \eta\rho\gamma R\left(\tau\right); \tag{6}$$

$$\lambda_1 \frac{\partial T}{\partial x}\bigg|_{x=L_x} = -\beta_1\left(T_{oc} - T\left(L_x, y, z, \tau\right)\right) - \eta\rho\gamma R\left(\tau\right); \tag{7}$$

$$\lambda_1 \frac{\partial T}{\partial y}\bigg|_{y=0} = -\beta_1\left(T_{oc} - T\left(x, 0, z, \tau\right)\right) - \eta\rho\gamma R\left(\tau\right); \tag{8}$$

$$\lambda_1 \frac{\partial T}{\partial y}\bigg|_{y=L_y} = -\beta_1\left(T_{oc} - T\left(x, L_y, z, \tau\right)\right) - \eta\rho\gamma R\left(\tau\right); \tag{9}$$

$$\frac{\partial T}{\partial z}\bigg|_{z=0} = 0; \tag{10}$$

$$\lambda_1 \frac{\partial T}{\partial z}\bigg|_{z=L_z} = -\beta_1\left(T_{oc} - T\left(x, y, L_z, \tau\right)\right) - \eta\rho\gamma R\left(\tau\right); \tag{11}$$

$$\lambda_2 \frac{\partial u}{\partial x}\bigg|_{x=0} = -\beta_2\left(u_{oc} - u\left(0, y, z, \tau\right)\right); \tag{12}$$

$$\lambda_2 \frac{\partial u}{\partial x}\bigg|_{x=L_x} = -\beta_2\left(u_{oc} - u\left(L_x, y, z, \tau\right)\right); \tag{13}$$

$$\lambda_2 \frac{\partial u}{\partial y}\bigg|_{y=0} = -\beta_2 \left(u_{oc} - u\left(x, 0, z, \tau\right)\right); \tag{14}$$

$$\lambda_2 \frac{\partial u}{\partial y}\bigg|_{y=L_y} = -\beta_2 \left(u_{oc} - u\left(x, L_y, z, \tau\right)\right); \tag{15}$$

$$\frac{\partial u}{\partial z}\bigg|_{z=0} = 0; \tag{16}$$

$$\lambda_2 \frac{\partial u}{\partial z}\bigg|_{z=L_z} = -\beta_2 \left(u_{oc} - u\left(x, y, L_z, \tau\right)\right). \tag{17}$$

Here $T(x, y, z, \tau)$ – is the temperature at the point $(x, y, z) \in \Omega$ at time $\tau \geqslant 0$; $u(x, y, z, \tau)$ – change of moisture over time; $a(x, y, z)$ – coefficient of thermal diffusivity; $\delta(x, y, z)$ – coefficient of moisture conductivity; $\nabla = \left\{\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right\}$ – nabla operator; $f\left(x, y, z, \tau\right) = b \cdot e^{-\alpha\tau}$ – is the intensity of the internal heat release of the mass; $b = \frac{u}{c_1}$ – heat release coefficient; c_1 – specific heat capacity; α – empirical parameter; $q\left(x, y, z, \tau\right) = \rho m_0 e^{-\xi\tau}$ – the intensity of internal sources of moisture, at constant values of the density of the material – ρ; ξ – drying coefficient; m_0 – maximum evaporation rate; β_1 – heat transfer coefficient; T_{oc} – ambient temperature; η – coefficients for carrying out the boundary condition to the dimensional form; γ – absorption coefficient; $R\left(\tau\right)$ – flux of solar radiation; β_2 – coefficient of moisture return; u_{oc} – ambient humidity.

The processes of heat and moisture transfer in heterogeneous porous media during the storage and drying of bodies can be studied, observed, and predicted using a mathematical model of this kind. This model accounts for the heterogeneity of the medium, heat and moisture exchange with the environment, daily variation in solar radiation, and internal heat and moisture release of the porous body.

3 Solution method

As the issue description makes clear, the target of research is characterized by a system of partial differential equations with a source of heat and moisture release, making an analytical solution challenging. In light of the aforementioned, for solving problems (3) through (17) we used the finite difference approach with the second order of precision in time and space variables, replacing the region of the continuous solution with a grid.

Let's introduce the space-time grid:

$$\Omega_{xyz\tau} = \left\{\left(x_i = i\Delta x,\ y_j = j\Delta y,\ z_k = k\Delta z,\ \tau_n = n\ \Delta\tau\right); \right.$$
$$\left. i = \overline{1, N_x};\ \ j = \overline{1, M_y},\ \ k = \overline{1, L_z},\ n = \overline{0, N_\tau},\ \ \Delta\tau = 1/N_\tau\right\}$$

and replace the differential operators of equation (3) with difference operators in Ox:

$$\frac{1}{2}\frac{T_{i,j,k}^{n+\frac{1}{3}} - T_{i,j,k}^{n}}{\Delta\tau/3} + \frac{1}{2}\frac{T_{i+1,j,k}^{n+\frac{1}{3}} - T_{i+1,j,k}^{n}}{\Delta\tau/3} =$$
$$= \frac{a_{i+0,5,j,k} T_{i+1,j,k}^{n+\frac{1}{3}} - \left(a_{i+0,5,j,k} + a_{i-0,5,j,k}\right) T_{i,j,k}^{n+\frac{1}{3}} + a_{i-0,5,j,k} T_{i-1,j,k}^{n+\frac{1}{3}}}{\Delta x^2} +$$
$$+ \frac{a_{i,j+0,5,k} T_{i,j+1,k}^{n} - \left(a_{i,j+0,5,k} + a_{i,j-0,5,k}\right) T_{i,j,k}^{n} + a_{i,j-0,5,k} T_{i,j-1,k}^{n}}{\Delta y^2} +$$
$$+ \frac{a_{i,j,k+0,5} T_{i,j,k+1}^{n} - \left(a_{i,j,k+0,5} + a_{i,j,k-0,5}\right) T_{i,j,k}^{n} + a_{i,j,k-0,5} T_{i,j,k-1}^{n}}{\Delta z^2} +$$

$$+\frac{a_{i+0,5,j,k}u^n_{i+1,j,k}-\left(a_{i+0,5,j,k}+a_{i-0,5,j,k}\right)u^n_{i,j,k}+a_{i-0,5,j,k}u^n_{i-1,j,k}}{\Delta x^2}+$$
$$+\frac{a_{i,j+0,5,k}u^n_{i,j+1,k}-\left(a_{i,j+0,5,k}+a_{i,j-0,5,k}\right)u^n_{i,j,k}+a_{i,j-0,5,k}u^n_{i,j-1,k}}{\Delta y^2}+ \tag{18}$$
$$+\frac{a_{i,j,k+0,5}u^n_{i,j,k+1}-\left(a_{i,j,k+0,5}+a_{i,j,k-0,5}\right)u^n_{i,j,k}+a_{i,j,k-0,5}u^n_{i,j,k-1}}{\Delta z^2}+\frac{1}{3}f^{n+\frac{1}{3}}_{i,j,k}.$$

Grouping like terms, we obtain a system of tridiagonal algebraic equations:

$$a_{T,i,j,k}T^{n+\frac{1}{3}}_{i-1,j,k}-b_{T,i,j,k}T^{n+\frac{1}{3}}_{i,j,k}+c_{T,i,j,k}T^{n+\frac{1}{3}}_{i+1,j,k}=-d_{T,i,j,k}. \tag{19}$$

Further, we approximate the boundary condition (6) with respect to Ox and obtain:

$$\lambda_1\frac{-3T^{n+\frac{1}{3}}_{0,j,k}+4T^{n+\frac{1}{3}}_{1,j,k}-T^{n+\frac{1}{3}}_{2,j,k}}{2\Delta x}=-\beta_1T_{oc}+\beta_1T^{n+\frac{1}{3}}_{0,j,k}-\varphi^{n+\frac{1}{3}}, \tag{20}$$

where $\varphi=\eta\rho\gamma R\left(\tau\right)$.
From the system of equations (19), when $i=1$, we get

$$a_{T,1,j,k}T^{n+\frac{1}{3}}_{0,j,k}-b_{T,1,j,k}T^{n+\frac{1}{3}}_{1,j,k}+c_{T,1,j,k}T^{n+\frac{1}{3}}_{2,j,k}=-d_{T,1,j,k}. \tag{21}$$

Putting $T^{n+\frac{1}{3}}_{2,j,k}$ from (21) into (20), we find $T^{n+\frac{1}{3}}_{0,j,k}$:

$$T^{n+\frac{1}{3}}_{0,j,k}=\alpha_{T,0,j,k}T^{n+\frac{1}{3}}_{1,j,k}+\beta_{T,0,j,k}, \tag{22}$$

where the sweep coefficients $\alpha_{T,0,j,k}$, $\beta_{T,0,j,k}$ are calculated using the formulas:

$$\alpha_{T,0,j,k}=\frac{\lambda_1b_{T,1,j,k}-4\lambda_1c_{T,1,j,k}}{a_{T,1,j,k}\lambda_1-3c_{T,1,j,k}\lambda_1-\ 2\Delta xc_{T,1,j,k}\beta_1};$$
$$\beta_{T,0,j,k}=\frac{-d_{T,1,j,k}\lambda_1-2\Delta xc_{T,1,j,k}\beta_1T_{oc}-2\Delta xc_{T,1,j,k}\varphi^{n+\frac{1}{3}}}{a_{T,1,j,k}\lambda_1-3c_{T,1,j,k}\lambda_1-\ 2\Delta xc_{T,1,j,k}\beta_1}.$$

Similarly, approximating the boundary condition (7) with respect to Ox, we obtain:

$$\lambda_1\frac{T^{n+\frac{1}{3}}_{N-2,j,k}-4T^{n+\frac{1}{3}}_{N-1,j,k}+3T^{n+\frac{1}{3}}_{N,j,k}}{2\Delta x}=-\beta_1T_{oc}+\beta_1T^{n+\frac{1}{3}}_{N,j,k}-\varphi^{n+\frac{1}{3}}. \tag{23}$$

where $\varphi=\eta\rho\gamma R\left(\tau\right)$.

Applying the sweep method for the sequence at $N, N-1$ and $N-2$, we find $T^{n+\frac{1}{3}}_{N-1,j,k}$ and $T^{n+\frac{1}{3}}_{N-2,j,k}$:

$$T^{n+\frac{1}{3}}_{N-1,j,k}=\alpha_{T,N-1,j,k}T^{n+\frac{1}{3}}_{N,j,k}+\beta_{T,N-1,j,k}; \tag{24}$$

$$T^{n+\frac{1}{3}}_{N-2,j,k}=\alpha_{T,N-2,j,k}\alpha_{T,N-1,j,k}T^{n+\frac{1}{3}}_{N,j,k}+\alpha_{T,N-2,j,k}\beta_{T,N-1,j,k}+\beta_{T,N-2,j,k}. \tag{25}$$

Putting $T^{n+\frac{1}{3}}_{N-1,j,k}$ from (24) and $T^{n+\frac{1}{3}}_{N-2,j,k}$ from (25) to (23), we find $T^{n+\frac{1}{3}}_{N,j,k}$:

$$T^{n+\frac{1}{3}}_{N,j,k}=\frac{-\lambda_1\alpha_{T,N-2,j,k}\beta_{T,N-1,j,k}-\lambda_1\beta_{T,N-2,j,k}+4\lambda_1\beta_{T,N-1,j,k}-2\Delta x\beta_1T_{oc}-2\Delta x\varphi^{n+\frac{1}{3}}}{3\lambda_1-2\Delta x\beta_1+\lambda_1\alpha_{T,N-2,j,k}\alpha_{T,N-1,j,k}-4\lambda_1\alpha_{T,N-1,j,k}}. \tag{26}$$

The values of the temperature sequence $T_{N-1,j,k}^{n+\frac{1}{3}}$, $T_{N-2,j,k}^{n+\frac{1}{3}}$, ..., $T_{1,j,k}^{n+\frac{1}{3}}$ are determined by the method of back-sweep by decreasing i:

$$T_{i,j,k}^{n+\frac{1}{3}} = \alpha_{T,i,j,k} T_{i+1,j,k}^{n+\frac{1}{3}} + \beta_{T,i,j,k}, \quad i = \overline{N-1,\, 1},\ j = \overline{0,\, M},\ k = \overline{0,\, L}. \tag{27}$$

Similarly, equation (4) is approximated by Ox finite difference relations and grouping similar terms, we obtain a system of tridiagonal algebraic equations with respect to the required variables:

$$a_{u,i,j,k}\, u_{i-1,j,k}^{n+\frac{1}{3}} - b_{u,i,j,k}\, u_{i,j,k}^{n+\frac{1}{3}} + c_{u,i,j,k}\, u_{i+1,j,k}^{n+\frac{1}{3}} = -d_{u,i,j,k}. \tag{28}$$

Further, we approximate the boundary condition (12) with the second order of accuracy in Ox and obtain:

$$\lambda_2 \frac{-3u_{0,j,k}^{n+\frac{1}{3}} + 4u_{1,j,k}^{n+\frac{1}{3}} - u_{2,j,k}^{n+\frac{1}{3}}}{2\Delta x} = -\beta_2 u_{oc} + \beta_2 u_{0,j,k}^{n+\frac{1}{3}}. \tag{29}$$

From the system of equations (28), for $i = 1$, we get:

$$a_{u,1,j,k}\, u_{0,j,k}^{n+\frac{1}{3}} - b_{u,1,j,k}\, u_{1,j,k}^{n+\frac{1}{3}} + c_{u,1,j,k}\, u_{2,j,k}^{n+\frac{1}{3}} = -d_{u,1,j,k}. \tag{30}$$

Putting $u_{2,j,k}^{n+\frac{1}{3}}$ from (30) into (29), we find the value of $u_{0,j,k}^{n+\frac{1}{3}}$:

$$u_{0,j,k}^{n+\frac{1}{3}} = \alpha_{u,0,j,k}\, u_{1,j,k}^{n+\frac{1}{3}} + \beta_{u,0,j,k}. \tag{31}$$

From relation (31), the sweep coefficients are determined as:

$$c\alpha_{u,0,j,k} = \frac{\lambda_2 b_{u,1,j,k} - 4\lambda_2 c_{u,1,j,k}}{a_{u,1,j,k}\lambda_2 - 3c_{u,1,j,k}\lambda_2 - \ 2\Delta x c_{u,1,j,k}\beta_2};$$
$$\beta_{u,0,j,k} = \frac{-d_{u,1,j,k}\lambda_2 - 2\Delta x c_{u,1,j,k}\beta_2 u_{oc}}{a_{u,1,j,k}\lambda_2 - 3c_{u,1,j,k}\lambda_2 - \ 2\Delta x c_{u,1,j,k}\beta_2}.$$

Similarly, approximating the boundary condition (13) with respect to Ox, we obtain:

$$\lambda_2 \frac{u_{N-2,j,k}^{n+\frac{1}{3}} - 4u_{N-1,j,k}^{n+\frac{1}{3}} + 3u_{N,j,k}^{n+\frac{1}{3}}}{2\Delta x} = -\beta_2 u_{oc} + \beta_2 u_{N,j,k}^{n+\frac{1}{3}}. \tag{32}$$

Applying the sweep method for the sequence $N, N-1$ and $N-2$, we find $u_{N-1,j,k}^{n+\frac{1}{3}}$ and $u_{N-2,j,k}^{n+\frac{1}{3}}$:

$$u_{N-1,j,k}^{n+\frac{1}{3}} = \alpha_{u,N-1,j,k} u_{N,j,k}^{n+\frac{1}{3}} + \beta_{u,N-1,j,k}; \tag{33}$$

$$u_{N-2,j,k}^{n+\frac{1}{3}} = \alpha_{u,N-2,j,k}\alpha_{u,N-1,j,k}\, u_{N,j,k}^{n+\frac{1}{3}} + \alpha_{u,N-2,j,k}\beta_{u,N-1,j,k} + \beta_{u,N-2,j,k}. \tag{34}$$

Putting $u_{N-1,j,k}^{n+\frac{1}{3}}$ from (33) and $u_{N-2,j,k}^{n+\frac{1}{3}}$ from (34) to (32), we find $u_{N,j,k}^{n+\frac{1}{3}}$:

$$u_{N,j,k}^{n+\frac{1}{3}} = \frac{-\lambda_2\alpha_{u,N-2,j,k}\beta_{u,N-1,j,k} - \lambda_2\beta_{u,N-2,j,k} + 4\lambda_2\beta_{u,N-1,j,k} - 2\Delta x\beta_2 u_{oc}}{3\lambda_2 - 2\Delta x\beta_2 + \lambda_2\alpha_{u,N-2,j,k}\alpha_{u,N-1,j,k} - 4\lambda_2\alpha_{u,N-1,j,k}}.$$

The values of the moisture sequence $u_{N-1,j,k}^{n+\frac{1}{3}}$, $u_{N-2,j,k}^{n+\frac{1}{3}}$, ..., $u_{1,j,k}^{n+\frac{1}{3}}$are determined by the back-sweep method to decrease i:

$$u_{i,j,k}^{n+\frac{1}{3}} = \alpha_{u,i,j,k}\, u_{i+1,j,k}^{n+\frac{1}{3}} + \beta_{u,i,j,k}\,, \text{ where } i = \overline{N-1,\, 1},\; j = \overline{0,\, M},\; k = \overline{0,\, L}.$$

Further, a similar action is performed by on $n + \frac{2}{3}$ time layer, and then by Oz on $n+1$ time layer, we find $T_{i,j,L}^{n+1}$:

$$T_{i,j,L}^{n+1} = \frac{-\lambda_1 \overline{\overline{\alpha}}_{T,i,j,L-2} \overline{\overline{\beta}}_{T,i,j,L-1} - \lambda_1 \overline{\overline{\beta}}_{T,i,j,L-2} + 4\lambda_1 \overline{\overline{\beta}}_{T,i,j,L-1} - 2\Delta z \beta_1 T_{oc} - 2\Delta z \varphi^{n+1}}{3\lambda_1 - 2\Delta z \beta_1 + \lambda_1 \overline{\overline{\alpha}}_{T,i,j,L-2}\, \overline{\overline{\alpha}}_{T,i,j,L-1} - 4\lambda_1 \overline{\overline{\alpha}}_{T,i,j,L-1}}.$$

$$T_{i,j,k}^{n+1} = \overline{\overline{\alpha}}_{T,i,j,k}\, T_{i,j,k+1}^{n+1} + \overline{\overline{\beta}}_{T,i,j,k}\,,$$

$$i = \overline{0, N},\; j = \overline{0, M},\; k = \overline{L-1,\, 1}.$$

Accordingly, we get $u_{i,j,L}^{n+1}$:

$$u_{i,j,L}^{n+1} = \frac{-\lambda_2 \overline{\overline{\alpha}}_{u,i,j,L-2} \overline{\overline{\beta}}_{u,i,j,L-1} - \lambda_2 \overline{\overline{\beta}}_{u,i,j,L-2} + 4\lambda_2 \overline{\overline{\beta}}_{u,i,j,L-1} - 2\Delta z \beta_2 u_{oc}}{3\lambda_2 - 2\Delta z \beta_2 + \lambda_2 \overline{\overline{\alpha}}_{u,i,j,L-2} \overline{\overline{\alpha}}_{u,i,j,L-1} - 4\lambda_2 \overline{\overline{\alpha}}_{u,i,j,L-1}}.$$

$$u_{i,j,k}^{n+1} = \overline{\overline{\alpha}}_{u,i,j,k}\, u_{i,j,k+1}^{n+1} + \overline{\overline{\beta}}_{u,i,j,k}, \text{ where } i = \overline{0, N},\; j = \overline{0,\, M},\; k = \overline{L-1,\, 1}.$$

In order to handle the three-dimensional problem of coupled heat and moisture transfer during the storage and drying of an inhomogeneous porous body, an efficient stable numerical solution based on the high-order precision finite-difference approach has been devised.

4 Results

Based on the proposed mathematical model and numerical algorithm, the object-oriented software program "HMTCotton"was built in $C\#$ to monitor and predict the processes of simultaneous heat and moisture transfer during the drying and storage of raw cotton in open areas.

Since the harvesting of cotton is seasonal, this raw cotton is stored in open space for a certain period. Raw cotton is used to form rectangular parallelepiped, covered with a tarpaulin on top. To enhance ventilation, form a through-the-hole. With long-term storage, the commercial quality of raw cotton is lost, and there are cases of its spontaneous combustion. In this regard, temperature and moisture prediction and determination of the maximum allowable storage periods are essential to the storage technology of raw cotton.

Computational experiments were carried out for September 2023 in the Bukhara region, where the average ambient temperature was +25°C (+36°C during the day, +18°C at night), and the average ambient humidity was 36% (28% during the day, 44% at night).

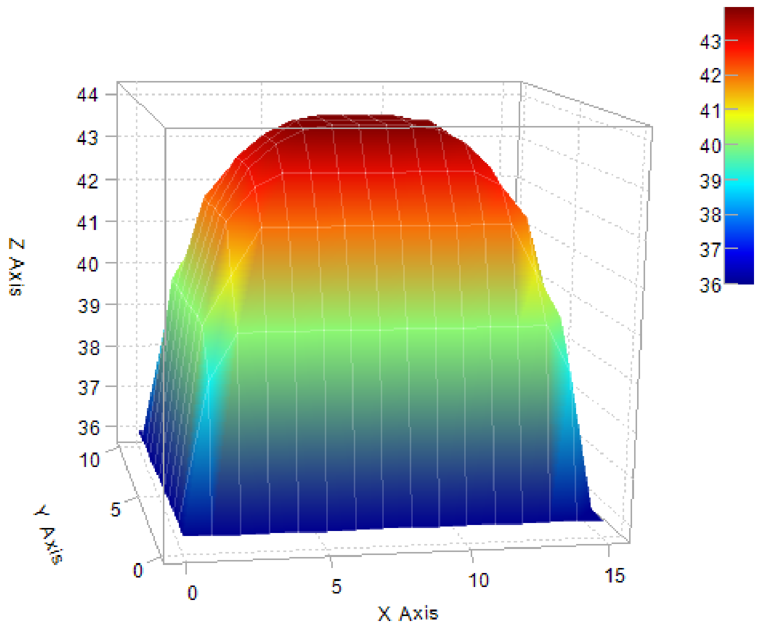

Figure 1 Change in temperature in the riot of raw cotton in three dimensions after 10 days of storage. $T_{oc} = 35^0 C,\quad T(x, y, z, 0) = 43^0 C,\ u_{oc} = 36\%,\ u(x, y, z, 0) = 44\%$

Figures 2–5 show layers that show the results of numerical calculations performed on a computer to give a visual representation of temperature and moisture changes in raw cotton.

Numerical experiments were carried out at various values of thermal diffusivity, moisture conductivity, various values of humidity and temperature of cotton riot including its properties. The size of the riot of raw cotton is taken as $L_x = 12\ m;\ \ L_y = 15\ m;\ \ L_z = 8\ m$. The cotton riot is erected so that the large sides of the rectangle are parallel to the north-south lines.

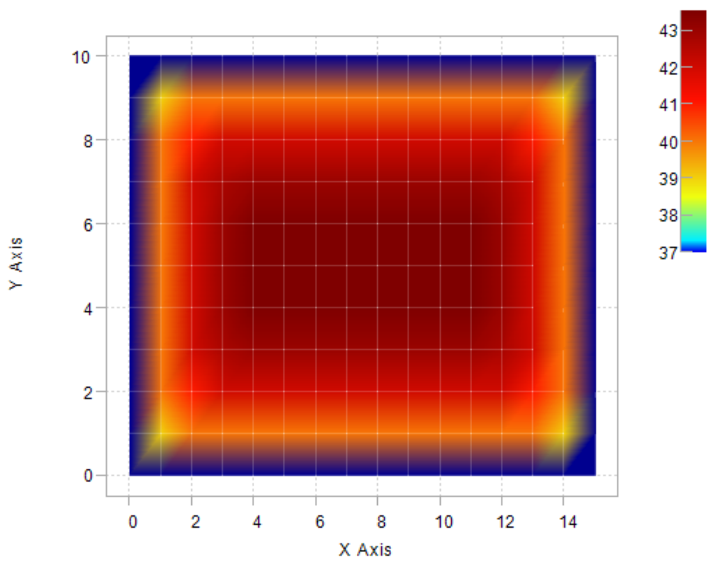

Figure 2 Shows the temperature change in a riot of raw cotton after 10 days in a layer at $z = 5m$. $T_{oc} = 37^0 C,\ T(x, y, z, 0) = 45^0 C,\ \ u_{oc} = 36\%,\ \ u(x, y, z, 0) = 42\%$

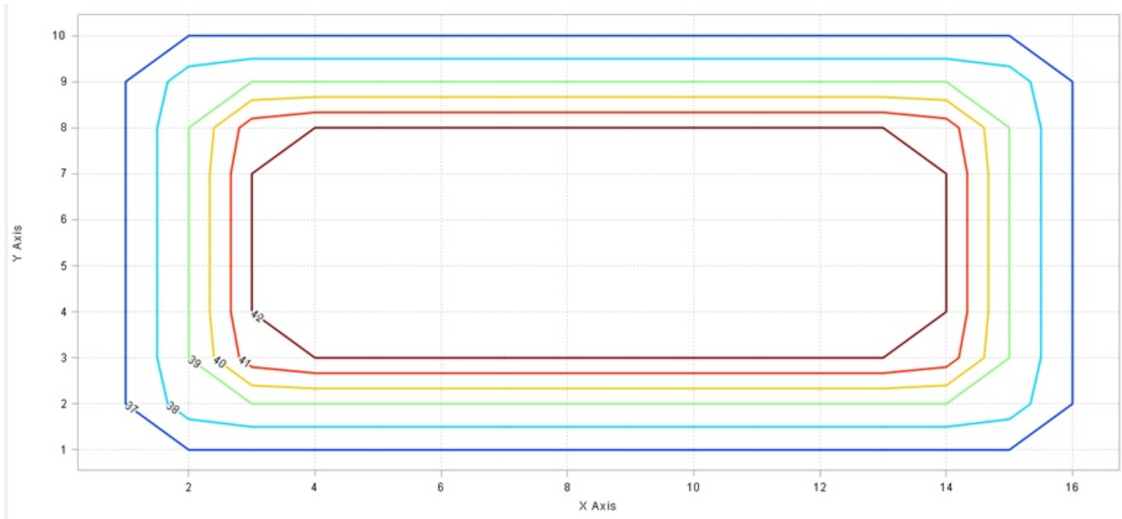

Figure 3 Shows the humidity change in a riot of raw cotton after 10 days in a layer at $z = 5m$. $T_{oc} = 37^0C$, $T(x, y, z, 0) = 45^0C$, $u_{oc} = 36\%$, $u(x, y, z, 0) = 42\%$

Figures 2-3 display the findings of the computational experiments on how temperature and moisture change along a plane based on x and y. The temperature climbs to 43°C over time, while the humidity inside the cotton remains at 42%.

After 30 days of storage, because of internal heat and moisture release, the cotton temperature rises to 52°C and its humidity reaches 55% (Fig. 4-5). Due to the increase of humidity and temperature in the interior part of the cotton, in practice, wet air is pumped out of the raw cotton using a ventilation unit.

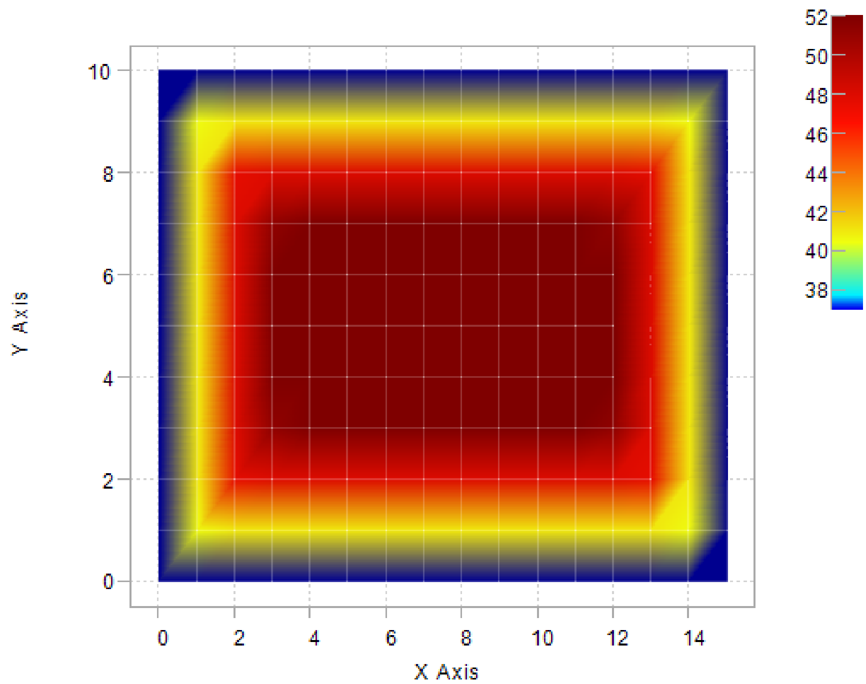

Figure 4 Shows the temperature change in a riot of raw cotton after 30 days in a layer at $z = 6m$. $T_{oc} = 35^0C$, $T(x, y, z, 0) = 52^0C$, $u_{oc} = 36\%$, $u(x, y, z, 0) = 42\%$

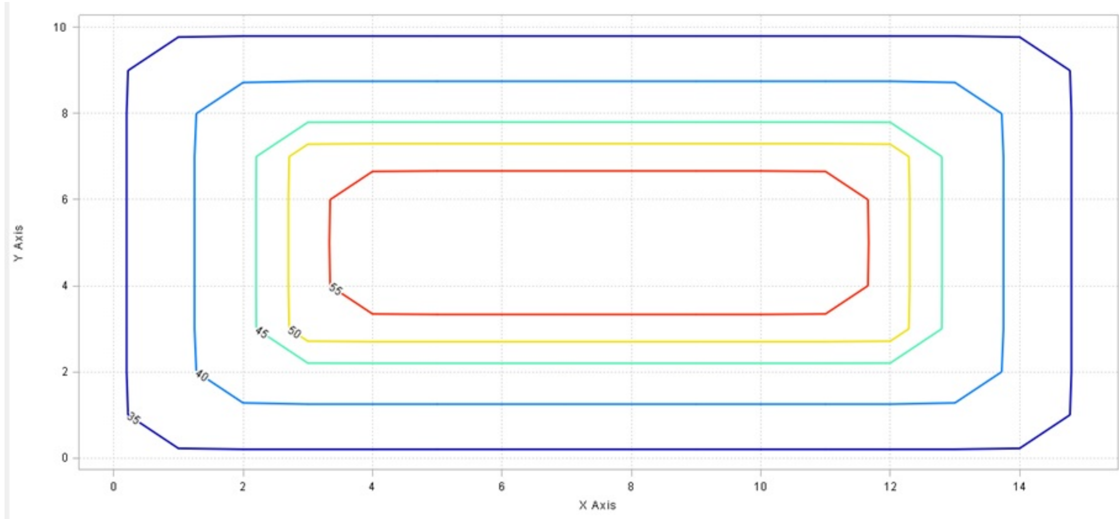

Figure 5 Shows the humidity change in a riot of raw cotton after 30 days in a layer at $z = 6m$. $T_{oc} = 35^0C$, $T\,(x, y, z, 0) = 52^0C$, $u_{oc} = 36\%$, $u\,(x, y, z, 0) = 42\%$

In October, when the average ambient temperature was +20°C (+27°C during the day, +13°C at night), and the average ambient humidity was 53% (44% during the day, 62% at night), wet air influences to the surface of cotton no more then inside.

Computational investigations show that the intensification of heat transfer between the cotton riot and the surrounding atmospheric air causes the cotton riot peak temperature to climb by at least 17°C after 30 days. Because of this, the raw cotton riot's porous bulk can attain maximum temperatures of up to 71°C at certain regions (Fig.6-7), which is hot for modern storage techniques.

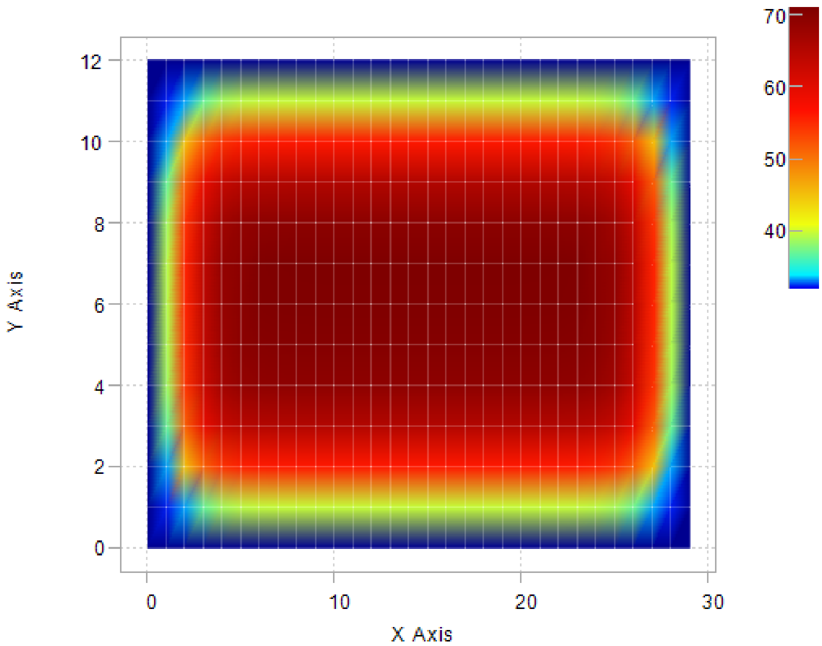

Figure 6 Shows the temperature change in a riot of raw cotton after 30 days in a layer at $z = 6m$. $T_{oc} = 35^0C$, $T\,(x, y, z, 0) = 52^0C$, $u_{oc} = 36\%$, $u\,(x, y, z, 0) = 42\%$

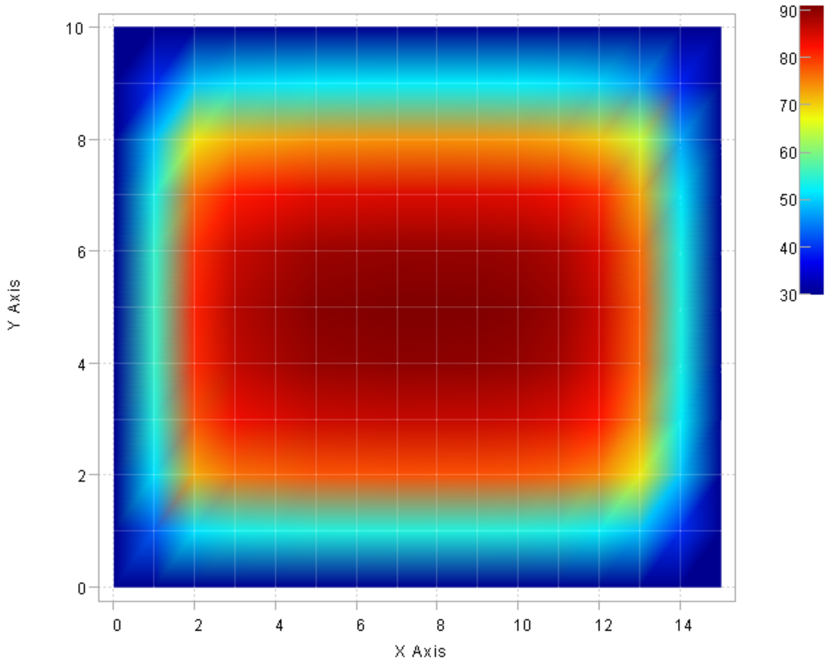

Figure 7 Shows the temperature change in a riot of raw cotton after 60 days in a layer at $z = 5m$. $T_{oc} = 29^0 C,\ \ T\,(x, y, z, 0) = 38^0 C,\ \ u_{oc} = 36\%,\ \ u\,(x, y, z, 0) = 42\%$

Raw cotton loses quality when it is stored for an extended period of time because key components of the fibre, such as density, moisture, contamination, oil content, and germination, change. In order to address the issues of storage and drying, there is a significant theoretical and practical requirement to understand how heat and moisture are distributed in raw cotton.

5 Conclusion

Multidimensional mathematical model, numerical algorithms, and software for handling the problem of simultaneous heat and moisture transfer in heterogeneous porous bodies have been developed to investigate, predict, and make management decisions about the drying and storage of raw cotton in open areas. Both the release of heat and moisture inside and the exchange of heat and moisture with the environment are considered by this model and algorithm. The results showed that when taking into account the initial moisture content of the raw cotton mass and the duration of storage, one cannot ignore the significance of internal heat and moisture release because these factors lead to debate and self-ignition, which have a significant negative impact on the quality of the cotton fiber.

According to experimental investigations, with prolonged storage (more than 50 days) in the open space when the temperature and relative humidity of raw cotton approach 91°C and 42% (Fig. 7), respectively, the quality of the cotton fibre starts to deteriorate. Additionally, it was noted that there were no quality deteriorations in the cotton with less then 45% moisture content that was stored in September and October (Fig. 1-6). The findings demonstrated that one cannot ignore the significance of internal heat and moisture release when considering the initial moisture content of the raw cotton mass and the length of storage because these factors cause debate and self-ignition, which have a significant negative impact on the quality of the cotton fibre.

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МНОГОМЕРНАЯ МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ОДНОВРЕМЕННОГО ТЕПЛО- И ВЛАГОПЕРЕНОСА ПРИ СУШКЕ И ХРАНЕНИИ ХЛОПКА-СЫРЦА НА ОТКРЫТЫХ ПЛОЩАДКАХ

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Предложена многомерная математическая модель одновременных процессов тепло- и влагопереноса в неоднородных пористых телах, учитывающая внутреннее тепло- и влаговыделение, тепло- и влагообмен с окружающей средой. На основе использования неявной конечно-разностной схемы второго порядка точности по временным и пространственным переменным создано эффективное численное решение задач. На основе разработанного численного алгоритма создано программное обеспечение для изучения и анализа процессов тепло- и влагопереноса при хранении и сушке хлопка-сырца на открытых площадках, позволяющее выявлять и прогнозировать изменения температуры и влажности в произвольных точках хлопка-сырца.

Ключевые слова: математическая модель, конечно-разностная схема, теплообмен, влагоперенос, хлопок-сырец.

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