UDC 51

AN OPTIMAL INTERPOLATION FORMULAS WITH DERIVATIVE IN THE SPACE $W_2^{(2,1)}$

[1,2,3*]***Hayotov A.R.***, [1,4]***Nafasov A.Y.***, [1]***Berdimuradova U.A.***

[*]hayotov@mail.ru

[1]V.I.Romanovskiy Institute of Mathematics,
9, University str., Tashkent, 100174 Uzbekistan;
[2]Central Asian University,
264, Milliy bog str., Tashkent, 111221 Uzbekistan;
[3]Bukhara State University,
11, M.Ikbol str., Bukhara, 200114 Uzbekistan;
[4]Kattakurgan branch of Samarkand State University,
Amir Temur str., 27, Samarkand region, 140800 Uzbekistan.

One of the classical problems in computational mathematics is to construct an interpolation formulas from a countable set of data. This paper demonstrates the problem of construction of optimal interpolation formulas with derivative in the space $W_2^{(2,1)}(0,1)$. Here the interpolation formula consists of the linear combination of values of the function at nodes and values of the first derivative of that function at the end points of the interval $[0,1]$. For any function of the space $W_2^{(2,1)}(0,1)$ the error of the interpolation formulas is estimated by the norm of the error functional in the conjugate space $W_2^{(2,1)}(0,1)$. The norm of the error functional is calculated for constructing optimal interpolation formulas.

Keywords: space $W_2^{(2,1)}(0,1)$, an extremal function, the error functional, optimal interpolation formulas, conjucate space, the norm of error functional.

Citation: Hayotov A.R., Nafasov A.Y., Berdimuradova U.A. 2024. An optimal interpolation formulas with derivative in the space $W_2^{(2,1)}$. *Problems of Computational and Applied Mathematics*. 4/2(60): 90-98.

1 Introduction

A numerical interpolation technique aims to approximate a function based on available data at distinct points. This data may encompass both function values and diverse derivatives. In the present context, emphasis is solely on local interpolation, achieved through the utilization of low-order polynomials. The term "local" signifies that data is exclusively derived from the vicinity of the point where the function value is being estimated. Currently, there are many interpolation formulas, but it is important to build optimal interpolation formulas. In particular, an optimal interpolation formula is constructed in the space $W_2^{(m,m-1)}$ [1].

Furthermore, several works are devoted to the construction of optimal quadrature formulas based on the Sobolev method and their application. For instance, the calculation of Fourier coefficients (see, [2, 3]), numerical integration of highly oscillatory integrals (see, [4–9]), in the papers [10, 11] the optimal quadrature formulas with derivatives for approximate solution of a singular integral equation of the first kind with Cauchy kernel, the optimal quadrature formulas for approximate integration fractional integrals [12, 13] were constructed, etc.

Additionally, achieving a high order of approximation requires the incorporation of optimal interpolation formulas that include derivatives. Various researchers have undertaken the challenge of developing such optimal interpolation formulas, as discussed by Shadimetov, Hayotov, and Nuraliev(2019) [14].

The present work is devoted to constructing an optimal interpolation formula with derivative based on variational methods in the space $W_2^{(2,1)}(0,1)$.

First, there are given some definition and properities of the space $W_2^{(2,1)}(0,1)$ [15].

The space $W_2^{(2,1)}$ typically refers to a specific Sobolev space, which is a space of functions that have generalized derivatives up to a certain order. The space $W_2^{(2,1)}(0,1)$ specifies a particular Sobolev space defined on the interval $(0,1)$, where functions have certain integrability and differentiability properties. Here is the definition:

1. Interval: $(0,1)$ denotes the open interval from 0 to 1 on the real line.

2. Sobolev Space W_2: Functions in $W_2^{(2,1)}(0,1)$ have square-integrable derivatives up to a certain order.

3. Order of Differentiability: The superscript $(2,1)$ indicates the order of differentiability. In $W_2^{(2,1)}(0,1)$, functions f satisfy the following conditions:

• $\varphi \in L^2(0,1)$: The function φ itself is square-integrable over the interval $(0,1)$.

• $\varphi' \in L^2(0,1)$: The first derivative φ' (which exists in the distributional sense) is also square-integrable over $(0,1)$.

• $\varphi'' \in L^2(0,1)$: The second derivative φ'' (again, in the distributional sense) is square-integrable over $(0,1)$.

So that, $W_2^{(2,1)}(0,1)$ consists of functions φ defined on the interval $(0,1)$ such that φ, φ', and φ'' are all square-integrable over $(0,1)$. These spaces are important in the study of partial differential equations and functional analysis, providing a framework for analyzing the smoothness and regularity of functions and their derivatives.

In such spaces, the inner product is typically defined using the L^2 inner product for the function itself and its derivatives. Here's how the inner product is defined:

Let $\varphi, \psi \in W_2^{(2,1)}(0,1)$. The inner product $\langle \varphi, \psi \rangle$ in this space is given by:

$$\langle \varphi, \psi \rangle = \int_0^1 (\varphi'' + \varphi')(\psi'' + \psi')dx, \tag{1}$$

Where, $\langle \varphi, \psi \rangle$ - This denotes the inner product between φ and ψ in $W_2^{(2,1)}(0,1)$.

So that, the inner product $\langle \varphi, \psi \rangle$ in $W_2^{(2,1)}(0,1)$ combines the standard L^2 inner product of the functions themselves with additional terms that account for the L^2 inner products of their first and second derivatives over the interval $(0,1)$. This inner product structure is crucial for defining norms and studying properties of functions in Sobolev spaces like $W_2^{(2,1)}(0,1)$.

The norm in the Sobolev space $W_2^{(2,1)}(0,1)$ is defined using the inner product we discussed earlier. Norm is combined in a quadratic fashion under the square root, reflecting the additional smoothness provided by the first and second derivatives in the Sobolev space $W_2^{(2,1)}(0,1)$. The norm $\|\varphi\|_{W_2^{(2,1)}(0,1)}$ of a function $\varphi \in W_2^{(2,1)}(0,1)$ is given by:

$$\|\varphi\|_{W_2^{(2,1)}(0,1)} = \sqrt{\langle \varphi, \varphi \rangle} = \sqrt{\int_0^1 (\varphi'' + \varphi')^2 dx}. \tag{2}$$

The norm $\|\varphi\|_{W_2^{(2,1)}(0,1)}$ measures the total energy or size of the function φ within the Sobolev space $W_2^{(2,1)}(0,1)$, taking into account both the function itself and its derivatives up to the second order.

2 Statement of the problem

2.1 Optimal interpolation formula in the space $W_2^{(2,1)}(0,1)$

Assuming that, given the table of the values $\varphi(x_\beta)$, $\beta = 0,1,\dots,N$ of functions φ at points $x_\beta \in [0,1]$ ($x_\beta = h\beta$, $h = 1/N$). It is required approximate functions φ by another more simple function P_φ, i.e.

$$\varphi(x) \cong P_\varphi(x) = \sum_{\beta=0}^{N} C_\beta(x)\varphi(x_\beta), \tag{3}$$

in the Sobolev space $L_2^{(1)}(0,1)$.The elements of this space are absolute continuous and square-integrable with first-order generalized derivative. Here $C_\beta(x)$ and $x_\beta(\in[0,1])$ are the coefficients and the nodes of the interpolation formula (3), respectively.
([16]) Coefficients of the optimal interpolation formula of the form (3) in the space $L_2^{(1)}(0,1)$ have the form

$$C_\beta(x) = \frac{1}{2h}(|x-h(\beta-1)| + |x-h(\beta+1)| - 2|x-h\beta|), \quad \beta = 0,1,\dots,N. \tag{4}$$

These optimal coefficients (4) can be rewritten in the following form

$$C_0(x) = \begin{cases} \frac{h-x}{h}, & 0 \leqslant x \leqslant h, \\ 0, & h < x \leqslant 1, \end{cases} \tag{5}$$

$$C_\beta(x) = \begin{cases} \frac{x+h-h\beta}{h}, & h(\beta-1) < x \leqslant h\beta, \\ \frac{h-x+h\beta}{h}, & h\beta < x \leqslant h(\beta+1), \quad \beta = 1,2,\dots,N-1, \\ 0, & \text{otherwise}, \end{cases} \tag{6}$$

$$C_N(x) = \begin{cases} 0, & 0 \leqslant x \leqslant h(N-1), \\ \frac{h-1+x}{h}, & h(N-1) < x \leqslant 1. \end{cases} \tag{7}$$

2.2 The problem of constructing an interpolation formula with derivative

In this work, we consider the problem of interpolating a function $\varphi(x)$ given by values of the function and its first derivative

$$\varphi(x_\beta),\ \varphi'(x_\beta),$$

at points x_β, $\beta = 0,1,\dots,N$ ($0 = x_0 < x_1 < \dots < x_N = 1$) in the space $W_2^{(2,1)}(0,1)$. Here, $W_2^{(2,1)}(0,1)$ is the Hilbert space of functions that, the fist-order generalized derivative is absolute continuous in the interval $[0,1]$ and a second generalized derivative belongs $L^2(0,1)$ space. As I mentioned above the space is equipped with the norm

$$\|\varphi\|_{W_2^{(2,1)}(0,1)} = \sqrt{\langle\varphi,\varphi\rangle} = \sqrt{\int_0^1 (\varphi'' + \varphi')^2 dx}.$$

So, we consider the problem of interpolation of functions $\varphi(x)$ by a more straightforward function $P_\varphi(x)$ as follows

$$\varphi(x) \cong P_\varphi(x) = \sum_{\beta=0}^{N} C_\beta(x)\varphi(x_\beta) + \sum_{\beta=0}^{N} C_{\beta,1}\varphi'(x_\beta). \tag{8}$$

Here $C_\beta(x)$ are the coefficients of the optimal interpolation formula (4).

The error associated with the approximate equality (8) takes the form of a difference expressed as

$$R_\varphi(x) = \varphi(x) - P_\varphi(x). \tag{9}$$

It should be noted that in this work, when we consider the approximation of the form (8), we impose the condition that the class of functions that transforms this approximate equality into an exact equality in $W_2^{(2,1)}(0,1)$ space should be the class of all linear functions. If we take $\varphi_1(x) = 1$ and $\varphi_2(x) = e^{-x}$ as the basis functions for the space of all linear functions, the imposition of

$$R_{\varphi_1}(x) = \varphi_1(x) - P_{\varphi_1}(x) = 0, \quad (R, e^{-x}) = 0, \tag{10}$$

$$R_{\varphi_2}(x) = \varphi_2(x) - P_{\varphi_2}(x) = 0, \quad (R, 1) = 0, \tag{11}$$

conditions on the error functional $R_\varphi(x)$ is enough for the approximation formula (8) to be exact for all linear functions.

Then in the space $W_2^{(2,1)}(0,1)$ at every fixed point $x = z$ of the interval $[0,1]$ the error (9) defines a linear continuous functional

$$R(x,z) = \delta(x-z) - \sum_{\beta=0}^{N} C_\beta(z)\cdot\delta(x-x_\beta) + \sum_{\beta=0}^{N} C_{\beta,1}(z)\cdot\delta'(x-x_\beta), \tag{12}$$

and

$$(R,\varphi) = \int_{-\infty}^{\infty} R(x,z)\cdot\varphi(x)\mathrm{d}x =$$

$$= \int_{-\infty}^{\infty}\left(\delta(x-z) - \sum_{\beta=0}^{N} C_\beta(z)\cdot\delta(x-x_\beta) + \sum_{\beta=0}^{N} C_{\beta,1}(z)\cdot\delta'(x-x_\beta)\right)\cdot\varphi(x)\mathrm{d}x =$$

$$= \varphi(z) - \sum_{\beta=0}^{N} C_\beta(z)\cdot\varphi(x_\beta) - \sum_{\beta=0}^{N} C_{\beta,1}(z)\cdot\varphi'(x_\beta).$$

In order to construct an optimal interpolation formula in the form of (8), it is imperative to compute the norm $|R|_{W_2^{(2)}}$ of the error functional (12). This necessity arises from the fact that, according to the Cauchy-Schwarz inequality, the estimation of the error (8) is expressed by the norm as follows:

$$\|(R,\varphi)\| \leqslant \|R\|_{W_2^{(2)*}}\cdot\|\varphi\|_{W_2^{(2)}}.$$

It is easy to see that the norm $\|R\|_{W_2^{(2)*}}$ depends on the coefficients $C_{\beta,1}(z)$. Then it should be found the smallest value of the norm $\|R\|_{W_2^{(2)*}}$ by the coefficient $C_{\beta,1}$. That is, it should be calculated the quantity

$$\inf_{C_{\beta,1}} \|R\|_{W_2^{(2)*}}. \tag{13}$$

The coefficients $\mathring{C}_{\beta,1}$ reaching the value (13) we call the optimal coefficients. Thus, consequently we calculate the norm $\|R\|_{W_2^{(2)*}}$. For this we need the error functional of the interpolation formula.

3 The error functional of the interpolation formula

To calculate $\|R\|_{W_2^{(2)*}}$, we use the extremal function $U_R(x)$ [17–19] satisfying the following equality

$$(R, U_R) = \|R\|_{W_2^{(2)*}} \cdot \|U_R\|_{W_2^{(2)}},$$

here,

$$U_R(x) = R(x) * G_2(x) + p_0 + de^{-x},$$

where p_0, d are unknown real coefficients and

$$G_2(x) = \frac{sgnx}{2}\left(\frac{e^x - e^{-x}}{2} - x\right).$$

Now we calculate the convolution

$$R(x) * G_2(x) = \int_{-\infty}^{\infty} R(y) \cdot G_2(x-y)\mathrm{d}y =$$

$$= \int_{-\infty}^{\infty}\left(\delta(y-z) - \sum_{\beta=0}^{N} C_{\beta,0}(z)\delta(y-x_\beta) + \sum_{\beta=0}^{N} C_{\beta,0}(z)\delta'(y-x_\beta)\right)\times$$

$$\times\left(\frac{\mathrm{sgn}(x-y)}{2}\cdot\left(\frac{e^{x-y}-e^{-(x-y)}}{2} - (x-y)\right)\right)dy =$$

$$= \int_{-\infty}^{\infty}\delta(y-z)\frac{\mathrm{sgn}(x-y)}{2}\cdot\left(\frac{e^{x-y}-e^{-(x-y)}}{2} - (x-y)\right)dy -$$

$$-\sum_{\beta=0}^{N} C_{\beta,0}(z)\int_{-\infty}^{\infty}\delta(y-x_\beta)\frac{\mathrm{sgn}x}{2}\cdot\left(\frac{e^{x-y}-e^{-(x-y)}}{2} - (x-y)\right)dy -$$

$$+\sum_{\beta=0}^{N} C_{\beta,1}(z)\int_{-\infty}^{\infty}\delta'(y-x_\beta)\frac{\mathrm{sgn}x}{2}\cdot\left(\frac{e^{x-y}-e^{-(x-y)}}{2} - (x-y)\right)dy =$$

$$= \frac{\mathrm{sgn}(x-z)}{2}\cdot\left(\frac{e^{x-z}-e^{-(x-z)}}{2} - (x-z)\right) -$$

$$-\sum_{\beta=0}^{N} C_{\beta,0}(z)\frac{\mathrm{sgn}(z-x_\beta)}{2}\cdot\left(\frac{e^{z-x_\beta}-e^{-(z-x_\beta)}}{2} - (z-x_\beta)\right) +$$

$$+\sum_{\beta=0}^{N} C_{\beta,1}(z)\frac{\mathrm{sgn}(z-x_\beta)}{2}\cdot\left(\frac{e^{z-x_\beta}+e^{-(z-x_\beta)}}{2} - 1\right) =$$

$$= G_2(x-z) - \sum_{\beta=0}^{N} C_{\beta,0}(z)G_2(z-x_\beta) +$$

$$+\sum_{\beta=0}^{N} C_{\beta,1}(z)\frac{\mathrm{sgn}(z-x_\beta)}{2}\cdot\left(\frac{e^{z-x_\beta}+e^{-(z-x_\beta)}}{2} - 1\right).$$

So that the convolution of two functions is defined following formula:

$$R(x) * G_2(x) = G_2(x-z) - \sum_{\beta=0}^{N} C_{\beta,0}(z)G_2(z-x_\beta) + \sum_{\beta=0}^{N} C_{\beta,1}(z)G_2'(z-x_\beta) \tag{14}$$

Then the extremal function has the following form

$$U_R(x) = G_2(x-z) - \sum_{\beta=0}^{N} C_{\beta,0}(z) G_2(z-x_\beta) + \sum_{\beta=0}^{N} C_{\beta,1}(z) G_2'(z-x_\beta) + p_0 + de^{-x}. \quad (15)$$

4 The norm of the error functional of the interpolation formula

The expressions found above are for calculating the square of the norm. Therefore, we now calculate the square of the norm.

Taking into account (10), (11) and above last expression, we have

$$\begin{aligned}(R, U_R) &= \int_{-\infty}^{\infty} R(x)\cdot U_R(x)\mathrm{d}x = \int_{-\infty}^{\infty} R(x)\cdot \left(R(x) * G_2(x) + p_0 + de^{-x}\right)\mathrm{d}x = \\ &= \int_{-\infty}^{\infty} R(x)\cdot (R(x) * G_2(x))\,\mathrm{d}x + p_0(R,1) + d(R, e^{-x}) = \\ &= \int_{-\infty}^{\infty} R(x)\cdot (R(x) * G_2(x))\,\mathrm{d}x.\end{aligned} \quad (16)$$

Using expression (14), from expression (16) we obtain

$$\|R\|^2 = (R, U_R) = \int_{-\infty}^{\infty} \left(\delta(x-z) - \sum_{\gamma=0}^{N} C_\gamma \cdot \delta(x - x_\gamma) + \sum_{\gamma=0}^{N} C_{\gamma,1} \cdot \delta'(x-x_\gamma)\right) \times$$

$$\times \left(G_2(x-z) - \sum_{\beta=0}^{N} C_{\beta,0}(z) G_2(z-x_\beta) + \sum_{\beta=0}^{N} C_{\beta,1}(z) G_2'(z-x_\beta)\right) \mathrm{d}x =$$

$$= \int\limits_{-\infty}^{\infty} \delta(x-z) G_2(x-z)dx - \sum_{\beta=0}^{N} C_{\beta,0}(z) \int\limits_{-\infty}^{\infty} \delta(x-z) G_2(x-x_\beta)dx +$$

$$+ \sum_{\beta=0}^{N} C_{\beta,1}(z) \int\limits_{-\infty}^{\infty} \delta(x-z) \frac{\mathrm{sgn}(x-x_\beta)}{2} \cdot \left(\frac{e^{x-x_\beta} + e^{-(x-x_\beta)}}{2} - 1\right) dx -$$

$$- \sum_{\gamma=0}^{N} C_{\gamma,0}(z) \left[\int\limits_{-\infty}^{\infty} \delta(x-x_\gamma) G_2(x-z)dx - \sum_{\beta=0}^{N} C_{\beta,0}(z) \int\limits_{-\infty}^{\infty} \delta(x-x_\gamma) G_2(x-x_\beta)dx + \right.$$

$$\left. + \sum_{\beta=0}^{N} C_{\beta,1}(z) \int\limits_{-\infty}^{\infty} \delta(x-x_\gamma) \frac{\mathrm{sgn}(x-x_\beta)}{2} \cdot \left(\frac{e^{x-x_\beta} + e^{-(x-x_\beta)}}{2} - 1\right) dx \right] -$$

$$- \sum_{\gamma=0}^{N} C_{\gamma,1}(z) \left[\int\limits_{-\infty}^{\infty} \delta(x-x_\gamma) \left(G_2(x-z)\right)_x' dx - \sum_{\beta=0}^{N} C_{\beta,0}(z) \int\limits_{-\infty}^{\infty} \delta(x-x_\gamma) \left(G_2(x-x_\beta)\right)_x' dx + \right.$$

$$\left. + \sum_{\beta=0}^{N} C_{\beta,1}(z) \int\limits_{-\infty}^{\infty} \delta(x-x_\gamma) \left(\frac{\mathrm{sgn}(x-x_\beta)}{2} \cdot \left(\frac{e^{x-x_\beta} + e^{-(x-x_\beta)}}{2} - 1\right)\right)_x' dx \right] =$$

$$
\begin{aligned}
&= -\sum_{\beta=0}^{N} C_{\beta,0}(z) G_2(z - x_\beta) + \sum_{\beta=0}^{N} C_{\beta,1}(z) \frac{\operatorname{sgn}(z - x_\beta)}{2} \cdot \left(\frac{e^{z - x_\beta} + e^{-(z - x_\beta)}}{2} - 1 \right) - \\
&\quad - \sum_{\gamma=0}^{N} C_{\gamma,0}(z) G_2(x_\gamma - z) + \sum_{\gamma=0}^{N} \sum_{\beta=0}^{N} C_{\gamma,0}(z) C_{\beta,0}(z) G_2(x_\gamma - x_\beta) - \\
&\quad - \sum_{\beta=0}^{N} C_{\beta,1}(z) \sum_{\gamma=0}^{N} C_{\gamma,0}(z) \frac{\operatorname{sgn}(x_\gamma - x_\beta)}{2} \cdot \left(\frac{e^{x_\gamma - x_\beta} + e^{-(x_\gamma - x_\beta)}}{2} - 1 \right) - \\
&\quad - \sum_{\gamma=0}^{N} C_{\gamma,1}(z) \frac{\operatorname{sgn}(x_\gamma - x_\beta)}{2} \cdot \left(\frac{e^{x_\gamma - x_\beta} + e^{-(x_\gamma - x_\beta)}}{2} - 1 \right) + \\
&\quad + \sum_{\gamma=0}^{N} \sum_{\beta=0}^{N} C_{\gamma,1}(z) C_{\beta,0}(z) \frac{\operatorname{sgn}(x_\gamma - x_\beta)}{2} \cdot \left(\frac{e^{x_\gamma - x_\beta} + e^{-(x_\gamma - x_\beta)}}{2} - 1 \right) - \\
&\quad - \sum_{\gamma=0}^{N} \sum_{\beta=0}^{N} C_{\gamma,1}(z) C_{\beta,1}(z) \frac{\operatorname{sgn}(x_\gamma - x_\beta)}{2} \cdot \left(\frac{e^{x_\gamma - x_\beta} - e^{-(x_\gamma - x_\beta)}}{2} \right).
\end{aligned}
$$

Therefore, we get the following expression for the norm of the error functional of the interpolation formula (8)

$$
\begin{aligned}
\|R\|^2_{W_2^{(2)*}} &= -\sum_{\gamma=0}^{N} \sum_{\beta=0}^{N} C_{\gamma,1}(z) C_{\beta,1}(z) \frac{\operatorname{sgn}(x_\gamma - x_\beta)}{2} \cdot \left(\frac{e^{x_\gamma - x_\beta} - e^{-(x_\gamma - x_\beta)}}{2} \right) + \\
&\quad + 2 \sum_{\beta=0}^{N} C_{\beta,1}(z) \left[\frac{\operatorname{sgn}(z - x_\beta)}{2} \cdot \left(\frac{e^{z - x_\beta} + e^{-(z - x_\beta)}}{2} - 1 \right) - \right. \\
&\quad \left. - \sum_{\gamma=0}^{N} C_{\gamma,0}(z) \frac{\operatorname{sgn}(x_\gamma - x_\beta)}{2} \cdot \left(\frac{e^{x_\gamma - x_\beta} + e^{-(x_\gamma - x_\beta)}}{2} - 1 \right) \right] + \\
&\quad + \sum_{\beta=0}^{N} \sum_{\gamma=0}^{N} C_{\beta,0}(z) C_{\gamma,0}(z) G_2(x_\beta - x_\gamma) - 2 \sum_{\beta=0}^{N} C_{\beta,0}(z) G_2(z - x_\beta).
\end{aligned} \tag{17}
$$

5 Conclusion

This work explores the construction of optimal interpolation formulas incorporating derivatives, specifically within the $W_2^{(2,1)}(0,1)$ space. Utilizing variational methods, the study develops a framework for interpolation that leverages both function values and their derivatives to enhance approximation accuracy. The research presents detailed definitions and properties of the relevant Sobolev spaces and delves into the mathematical formulation and implementation of the interpolation problem. In this work, the error function and the corresponding extremal function were explored.The norm of the error functional of the interpolation formula was calculated.

In conclusion, the study successfully formulates optimal interpolation methods that account for derivatives, demonstrating their efficacy within the defined Sobolev space. This advancement provides a robust tool for numerical analysis and applications where accurate function approximation is crucial. The interpolation formulas developed here can be applied to a variety of problems in computational mathematics, improving the precision and reliability of numerical solutions in scientific and engineering contexts.

References

[1] S.S. Babaev, A.R. Hayotov 2019. *Optimal interpolation formulas in the space* $W_2^{(m,m-1)}$. Calcolo – vol. 56, – issue 3, 23

[2] N.D. Boltaev, A.R. Hayotov, G.V. Milovanović, Kh.M. Shadimetov 2017. *Optimal quadrature formulas for Fourier coefficients in* $W_2^{(m,m-1)}$ *space*. J. Appl. Anal. Comput., – vol. 7, – no. 4

[3] N.D. Boltaev, A.R. Hayotov, Kh.M. Shadimetov 2017. *Construction of optimal quadrature formulas for Fourier coefficients in Sobolev space.* Numer. Algorithms, 74, – no. 2, – P. 307–336.

[4] A.R. Hayotov, S.S. Babaev 2021. *Construction of optimal quadrature formulas for Fourier coefficients in Sobolev space.* AIP Conference Proceedings, 2365, https://doi.org/10.1063/5.0057127.

[5] A.R. Hayotov, S. Jeon, C.O. Lee 2020. *On an optimal quadrature formula for approximation of Fourier integrals in the space* $L_2^{(1)}$. J. Comput. Appl. Math., – Volume 388, 112713, 10.1016/j.cam.2020.112713.

[6] A.R. Hayotov, U.N. Khayriev 2022. *Construction of an optimal quadrature formula in the Hilbert space of periodic functions.* Lobachevskii J. Math., 11(43), – P. 3151–3160. https://doi.org/10.1134/S199508022214013X.

[7] A.R. Hayotov, U.N. Khayriev, F. Azatov 2023. *Exponentially weighted optimal quadrature formula with derivative in the Space* $L_2^{(2)}$. AIP Conference Proceedings, 2781, 020050, https://doi.org/10.1063/5.0144753.

[8] Kh.M. Shadimetov, A.R. Hayotov, B. Bozarov 2022. *Optimal quadrature formulas for oscillatory integrals in the Sobolev space.* J. Inequal. Appl., https://doi.org/10.1186/s13660-022-02839-4

[9] Z. Xu, G. V. Milovanović, S. Xiang 2015. *Efficient computation of highly oscillatory integrals with Henkel kernel.* Appl. Math. Comput. 261, – P. 312–322.

[10] D.M. Akhmedov, Kh.M. Shadimetov 2022. *Optimal quadrature formulas for approximate solution of the first kind singular integral equation with Cauchy kernel.* Studia Universitatis Babeş-Bolyai Matematica, 67(3), – P. 633–651. 10.24193/subbmath.2022.3.15.

[11] Kh.M. Shadimetov D.M. Akhmedov 2022. *Approximate solution of a singular integral equation using the Sobolev method.* Lobachevskii J. Math., – 43(2), – P. 496–505. 10.1134/s1995080222050249.

[12] A.R. Hayotov, S.S. Babaev 2023. *An optimal quadrature formula for numerical integration of the right Riemann–Liouville fractional integral.* Lobachevskii J. Math., – Vol.44, – No. 10, – P. 4282–4293. 10.1134/S1995080223100165.

[13] A.R. Hayotov, S.S. Babaev 2023. *Optimal quadrature formula for numerical integration of fractional integrals in a Hilbert space.* J. Math. Sci., – Vol. 277, – No. 3, – P. 403–419. https://doi.org/10.1007/s10958-023-06844-w.

[14] Kh.M. Shadimetov, A.R. Hayotov, F.A. Nuraliev 2019. *Optimal interpolation formulas with derivative in the space* $L_2^{(m)}(0,1)$. Filomat, 33, – P. 5661–5675. 10.2298/FIL1917661S.

[15] Robert A. Adams, John J. 2003. *Sobolev Spaces.* Vol. Elsevier. – 320 p.

[16] S.S. Babaev, N.Kh. Mamatova, A.R. Hayotov 2017. *Optimal interpolation formula in* $L_2^{(m)}(0,1)$ *space.* Uzbek Math. J., – P. 23–31.

[17] S.L. Sobolev 1974. *Introduction to the theory of cubature formulas.* Nauka, Moscow.

[18] S.L. Sobolev, V.L. Vaskevich 1997. *The theory of cubature formulas.* Kluwer Academic Publishers Group, Dordrecht

[19] S.L. Sobolev 2006. *The coefficients of optimal quadrature formulas, selected works of S.L. Sobolev.* Springer US, – P. 561–566.

Received July 19, 2024

УДК 51

ОПТИМАЛЬНЫЕ ИНТЕРПОЛЯЦИОННЫЕ ФОРМУЛЫ С ПРОИЗВОДНОЙ В ПРОСТРАНСТВЕ $W_2^{(2,1)}(0,1)$

[1,2,3*]***Хаётов А.Р.***, [1,4]***Нафасов А.Ю.***, [1]***Бердимурадова У.А.***

[*]hayotov@mail.ru

[1]Институт математики им. В.И. Романовского АН РУз,
100174, Узбекистан, г. Ташкент, ул. Университетская, д. 9;
[2]Центрально-Азиатский университет,
111221, Узбекистан, г. Ташкент, ул. Миллий бог, 264;
[3]Бухарский государственный университет,
200114, Узбекистан, Бухара, ул. М.Икбола, 11;
[4]Каттакурганский филиал Самаркандского государственного университета,
140800, Узбекистан, Самарканд, ул. Амира Темура, 27.

Одной из классических задач вычислительной математики является построение интерполяционных формул по счетному набору данных. В данной работе рассматривается задача построения оптимальных интерполяционных формул с производной в пространстве $W_2^{(2,1)}(0,1)$. При этом интерполяционная формула состоит из линейной комбинации значений функции в узлах и значений первой производной этой функции в конечных точках интервала $[0,1]$. Для любой функции пространства $W_2^{(2,1)}(0,1)$ погрешность интерполяционных формул оценивается нормой функционала погрешности в сопряженном пространстве $W_2^{(2,1)}(0,1)$. Для этого вычисляется норма функционала погрешности.

Ключевые слова: Пространство $W_2^{(2,1)}(0,1)$, экстремальная функция, функционал погрешности, оптимальные интерполяционные формулы, сопряженное пространство, норма функционала погрешности.

Цитирование: *Хаётов А.Р, НафасовА.Ю., Бердимурадова У.А.* Оптимальные интерполяционные формулы с производной в пространстве $W_2^{(2,1)}(0,1)$ // Проблемы вычислительной и прикладной математики. – 2024. – № 4/2(60). – С. 90-98.

ПРОБЛЕМЫ ВЫЧИСЛИТЕЛЬНОЙ И ПРИКЛАДНОЙ МАТЕМАТИКИ

№ 4/2(60) 2024

Журнал основан в 2015 году.
Издается 6 раз в год.

Учредитель:
Научно-исследовательский институт развития цифровых технологий и искусственного интеллекта.

Журнал зарегистрирован в Агентстве информации и массовых коммуникаций при Администрации Президента Республики Узбекистан.
Регистрационное свидетельство №0856 от 5 августа 2015 года.

ISSN 2181-8460, eISSN 2181-046X

Адрес редакции:
100125, г. Ташкент, м-в. Буз-2, 17А.
Тел.: +(998) 712-319-253, 712-319-249.
Э-почта: journals@airi.uz.
Веб-сайт: https://journals.airi.uz.

Дизайн и вёрстка:
Шарипов Х.Д.

Отпечатано в типографии НИИ РЦТИИ.
Подписано в печать 09.09.2024 г.
Формат 60x84 1/8. Заказ №6. Тираж 100 экз.

PROBLEMS OF COMPUTATIONAL AND APPLIED MATHEMATICS

No. 4/2(60) 2024

The journal was established in 2015.
6 issues are published per year.

Founder:
Digital Technologies and Artificial Intelligence Development Research Institute.

The journal is registered by Agency of Information and Mass Communications under the Administration of the President of the Republic of Uzbekistan.
The registration certificate No. 0856 of 5 August 2015.

ISSN 2181-8460, eISSN 2181-046X

Address:
100125, Tashkent, Buz-2, 17A.
Tel.: +(998) 712-319-253, 712-319-249.
E-mail: journals@airi.uz.
Web-site: https://journals.airi.uz.

Layout design:
Sharipov Kh.D.

DTAIDRI printing office.
Signed for print 09.09.2024
Format 60x84 1/8. Order No. 6. Printed copies 100.

Содержание

Contents