UDC 519.644

APPROXIMATE SOLUTION FREDHOLM INTEGRAL EQUATION OF THE SECOND KIND BY THE OPTIMAL QUADRATURE METHOD

[1,2*]***Nuraliev F.A.***, [2,3]***Kuziev Sh.S.***, [4]***Djuraeva K.A.***
*f.nuraliev@mail.ru
[1]Tashkent International University,
7, Kichik khalka yoli str., Tashkent, 100084 Uzbekistan;;
[2]V.I.Romanovskiy Institute of Mathematics,
9, University str., Tashkent, 100174 Uzbekistan;
[3]Fergana Polytechnic Institute,
86, Fergana street, Fergana, 150100 Uzbekistan;
[4]University of exact and social sciences,
Karasaroy Street, 341a. Tashkent, 100069 Uzbekistan.

In the world, special attention is paid to the creation of various optimal calculation methods. This article is devoted to the construction of derivative optimal quadrature formulas in the space of differentiable functions using the Sobolev method. This quadrature formula consists of a linear combination of the values of the interval $[0, 1]$ up to the second derivative of the function at all nodes. The error of the quadrature formulas is estimated by the norm of the error function. We obtain the optimal quadrature formula by minimizing the norm of the error functional by the coefficients of the derivative quadrature formula. The resulting optimal quadrature formulas are exact for all degree functions. In addition, some methods for the numerical solution of Fredholm integral equations of the second type are given. These methods are derivative optimal quadrature formulas and Simpson's 1/3 method. Numerical examples are provided to demonstrate the effectiveness and accuracy of the work presented.

Keywords: Sobolev space, Fredholm integral equations, derivative optimal quadrature formula, error functional, Sobolev method, optimal coefficients.

Citation: Nuraliev F.A., Kuziev Sh.S., Djuraeva K.A. 2024. Approximate solution Fredholm integral equation of the second kind by the optimal quadrature method. *Problems of Computational and Applied Mathematics.* 4/2(60): 66-73.

1 Introduction. Statement problem

Numerical analysis is a branch of mathematics that deals with the development of effective methods for obtaining numerical solutions to complex problems. Due to the development of computing technology, numerical problem solving is developing. As a result, many practical software packages (Mathlab, Wolframalfa, Maple, C++, etc.) were created to solve complex problems effectively and easily. These applications solve problems using numerical methods, where the user can get results by entering the necessary variables without knowing the theoretical side of the numerical method. So, the question arises as to why we need to learn the number of methods when the application package is created. Here are some reasons to have a basic understanding of the theoretical foundations of numerical methods:

1. The study of numerical methods and their analysis allows the development of new numerical methods or, if the existing methods do not provide sufficient accuracy, it is necessary to choose another effective method.

2. There are many ways to solve problems, but it is important to choose the right method to achieve a clear result in a short time.

3. It is important to use numerical methods appropriately based on the specifics of the problem, and when the results are not as expected, it is important to understand what is going wrong in the program.

It is known that optimal quadrature formulas with derivative are methods of approximate calculation of definite integrals. They are necessary for calculating the integrals when an antiderivative of the functions under the integral cannot be expressed by elementary functions or the integrand exists only at discrete points. The use of formulas with a high level of algebraic accuracy in the integration of non-smooth functions does not lead to the good results. Therefore, it is important to construct optimal quadrature formulas with derivatives in the space of differentiable functions and estimate their errors. In this work, optimal quadrature formulas with derivatives are obtained based on the variational approach for the approximate calculation of definite integrals using the values of the given function up to the second order derivative at the nodes.

We consider the following quadrature formula with derivatives

$$\int\limits_0^1 u(x)dx \cong \sum_{\beta=0}^{N} C_0\,[\beta]u(h\beta) + \frac{h^2}{12}(u'(0)-u'(1)) + \sum_{\beta=0}^{N} C_1\,[\beta]u''(h\beta), \tag{1}$$

the difference below is called the error of the quadrature formula (1)

$$(\ell_N, u) = \int\limits_0^1 u(x)dx - \sum_{\beta=0}^{N} C_0\,[\beta]u(h\beta) - \frac{h^2}{12}(u'(0)-u'(1)) - \sum_{\beta=0}^{N} C_1\,[\beta]u''(h\beta)$$

, with error functional

$$\ell_N(x) = \varepsilon_{[0,1]}(x) - \sum_{\beta=0}^{N} C_0\,[\beta]\delta(x-h\beta) + \frac{h^2}{12}(\delta'(x)-\delta'(x-1)) - \sum_{\beta=0}^{N} C_1\,[\beta]\delta''(x-h\beta), \tag{2}$$

where $\varepsilon_{[0,1]}(x)$ is the indicator of the interval $[0,1]$, $\delta(x)$ is the Dirac delta-function, $C_0[\beta] = \begin{cases} \frac{h}{2}, & \beta = 0, N, \\ h, & \beta = \overline{1, N-1}, \end{cases}$ $C_1[\beta]$, $\beta = \overline{0, N}$ are unknown coefficients of the formula (1), $h = \frac{1}{N}$, N is a natural number.

The error functional $\ell_N(x)$ corresponding to the derived quadrature formula (1) is a linear, continuous functional defined in the conjugate space $L_2^{(m)*}(0,1)$.

According to the definition of the functional norm

$$\left\| \ell_N \left| L_2^{(m)^*} \right\| \right. = \sup_{\left\| u|L_2^{(m)} \right\| \neq 0} \frac{|(\ell_N, u)|}{\left\| u|L_2^{(m)} \right\|},$$

from this, using the definition of supremum, we get the Cauchy-Schwarz inequality

$$|(\ell_N, u| \leqslant \| \, u|L_2^{(m)} \, \| \cdot \| \, \ell_N|L_2^{(m)^*} \, \| \,.$$

As it can be seen from this inequality, the error of the derived quadrature formula (1) is estimated by the product of the norm of the error functional $\ell_N(x)$ obtained from the conjugate space $L_2^{(m)^*}$ and the norm of the function u obtained from the space $L_2^{(m)}$. When $\left\| u|\, L_2^{(m)} \right\| = 1$

$$\left\| \ell_N \left| L_2^{(m)^*} \right. \right\| = \sup_{\left\| u| L_2^{(m)} \right\|=1} |(\ell_N, u)| \,. \tag{3}$$

Thus, the evaluation of the error of the quadrature formula (1) on the $L_2^{(m)}(0,1)$ space elements (2) is related to the norm of the error functional ℓ_N in the $L_2^{(m)*}(0,1)$ conjugate space. ℓ_N The norm of the error function (1) depends on the coefficients and nodes of the quadrature formula. In this work, we solve the problem of minimizing the norm of the ℓ_N error functional only by coefficients when the nodes are fixed. In this work, a quadrature formula with derivatives is constructed using the Sobolev method based on the discrete analog of the d^{2m-4}/dx^{2m-4} differential operator in $L_2^{(m)}(0,1)$ space [1, 7, 9]. Since the error function ℓ_N is defined in space $L_2^{(m)}(0,1)$, it satisfies the following conditions

$$(\ell_N, x^\alpha) = 0, \quad \alpha = 0, 1, 2, ..., m-1. \tag{4}$$

Therefore, the condition $N \geqslant m-2$ must be fulfilled for quadrature formulas with derivatives of the form (1) to be exist. As noted above by the Cauchy-Schwarz inequality, the error of the quadrature formula (1) is estimated by the norm of the error functional (2). Moreover, the norm of the error function (2) depends on $C_1[\beta]$ coefficients, and we minimize the norm of the error function by $C_1[\beta]$ coefficients. $C_1[\beta]$ coefficients that satisfy equality (3) are called optimal coefficients and are denoted by $\overset{\circ}{C}{}_1[\beta]$, and the corresponding quadrature formula is called the *optimal quadrature formula.* Thus, in order to construct the optimal quadrature formula with derivatives in the form (1), we need to solve the following problems [1–3].

Problem 1. *Finding the general view of the error functional norm (2) of the quadrature formula with derivatives of the form (1) in $L_2^{(m)*}(0,1)$ space.*

Problem 2. *To find the coefficients that give the minimum to the norm of the error functional.*

We present the main results obtained in this work in the next section.

2 Main results

To solve **Problem 1**, we use the Riess theorem about the general form of a linear continuous functional and the concept of an extremal function. The norm of the error function of the quadrature formula can be written as follows [1–3, 10]:

$$\left\| \ell_N \left| L_2^{(m)^*}(0,1) \right. \right\|^2 = (\ell_N, U_\ell) = (\ell_N(x), ((-1)^m \ell_N(x) * G_{m-2}(x) + P_{m-1}(x))) \,,$$

where

$$U_\ell(x) = (-1)^m \ell_N(x) * G_{m-2}(x) + P_{m-1}(x),$$

$$G_{m-2}(x) = \frac{|x|^{2m-5}}{2 \cdot (2m-5)!},$$

is a solution of the equation

$$\frac{d^{2m-4}}{dx^{2m-4}} G_{m-2}(x) = \delta(x).$$

The following is true [8, 12].

Theorem 1. *An overview of the norm of the error functional (2) corresponding to the derivative quadrature formula (1) is as follows*

$$\left\| \ell_N \left| L_2^{(m)*} \right\| \right.^2 = (-1)^m \left[\sum_{\beta=0}^{N} C_1[\beta] \sum_{\gamma=0}^{N} C_1[\gamma] \frac{|h\beta - h\gamma|^{2m-5}}{2(2m-5)!} - 2\sum_{\beta=0}^{N} C_1[\beta] \int_0^1 \frac{|x-h\beta|^{2m-3}}{2(2m-3)!} dx - \right.$$

$$- \frac{h^2}{6} \sum_{\beta=0}^{N} C_1[\beta] \left[\frac{(h\beta)^{2m-4} + (1-h\beta)^{2m-4}}{2(2m-4)!} \right] + 2\sum_{\beta=0}^{N} C_0[\beta] \sum_{\gamma=0}^{N} C_1[\gamma] \frac{|h\beta - h\gamma|^{2m-3}}{2(2m-3)!} -$$

$$- \frac{h^2}{6} \sum_{\beta=0}^{N} C_0[\beta] \left[\frac{(h\beta)^{2m-2} + (1-h\beta)^{2m-2}}{2(2m-2)!} \right] - 2\sum_{\beta=0}^{N} C_0[\beta] \int_0^1 \frac{|x-h\beta|^{2m-1}}{2(2m-1)!} dx +$$

$$\left. + \sum_{\beta=0}^{N} C_0[\beta] \sum_{\gamma=0}^{N} C_0[\gamma] \frac{|h\beta - h\gamma|^{2m-1}}{2(2m-1)!} + \frac{h^2}{6(2m-1)!} + \frac{1}{(2m+1)!} + \frac{h^4}{144(2m-3)!} \right].$$

Theorem 2. *Coefficients of the optimal quadrature formula with derivatives in the form (1) in $L_2^{(m)}(0,1)$ space are determined as follows*

$$C_1[0] = h^3 \sum_{k=1}^{m-3} d_k \frac{q_k^N - q_k}{1-q_k},$$

$$C_1[\beta] = h^3 \sum_{k=1}^{m-3} d_k \left(q_k^\beta + q_k^{N-\beta} \right), \qquad \beta = \overline{1, N-1},$$

$$C_1[N] = h^3 \sum_{k=1}^{m-3} d_k \frac{q_k^N - q_k}{1-q_k},$$

where d_k satisfy the following system of $m-3$ linear equations

$$\sum_{k=1}^{m-3} d_k \sum_{i=1}^{l} \frac{q_k^{N+i} + (-1)^{i+1} q_k}{(1-q_k)^{i+1}} \Delta^i 0^l = \frac{B_{l+3}}{(l+1)(l+2)(l+3)}, \qquad l = \overline{1, m-3},$$

here B_{l+3} are Bernoulli numbers, $\Delta^i \gamma^l$ is the finite difference of order i of γ^j, q_k are the roots of the Euler-Frobenius polynomial $E_{2m-6}(q)$, $|q_k| < 1$.

In the next section, we analyze the obtained results by numerically solving Fredholm's integral equation of the second kind and comparing it with Simpson's 1/3 rule.

3 Numerical results

It is known that finding an analytical solution to integral equations is quite complicated. If the kernel of the integral equation consists of complex functions, solving the integral equation analytically requires a lot of work [4, 5]. We consider here the numerical solution of Fredholm's integral equation of the second kind, and in this connection, we recall the following method [6, 11].

We are given the integral equation

$$y(x) = \lambda \int_0^1 K(x,t) y(t) dt + f(x),$$

we replace the integral under the integral with the following sum

$$\int_0^1 K(x,t)y(t)dt \cong \sum_{j=0}^{N} \alpha_j K(x_i,t_j)y(t_j), \quad i=\overline{0,N},$$

where α_j are the coefficients of the quadrature formula. By inserting the last approximate equation into the given integral equation, we form a system of linear equations.

$$y_i - \lambda \sum_{j=0}^{N} \alpha_j K_{ij} y_j \bigtriangleup t = f_i, \qquad i=\overline{0,N},$$

where $x_i = t_i$, $f(x_i) = f_i$, $K(x_i,t_j) = K_{ij}$. We find y_i, $(i=\overline{1,N})$ from the system of linear equations.

We apply the derived optimal quadrature formula constructed in this work to the numerical solution of Fredholm's integral equation of the second kind. To do this, we first replace the first and second derivatives in the formula with finite differences.

$$\varphi^{'}(0) = \frac{-25\varphi(0)+48\varphi(h)-36\varphi(2h)+16\varphi(3h)-3\varphi(4h)}{12h}$$

$$\varphi^{'}(1) = \frac{25\varphi(1)-48\varphi((N-1)h)+36\varphi((N-2)h)-16\varphi((N-3)h)+3\varphi((N-4)h)}{12h}$$

$$\varphi''(x_i) = \frac{-\varphi(x_{i-2})+16\varphi(x_{i-1})-30\varphi(x_i)+16\varphi(x_{i+1})-\varphi(x_{i+2})}{12h^2}, \qquad i=\overline{2,N-2}$$

$$\varphi''(0) = \frac{45\varphi(0)-154\varphi(h)+214\varphi(2h)-156\varphi(3h)+61\varphi(4h)-10\varphi(5h)}{12h^2},$$

$$\varphi''(1) = \frac{1}{12h^2}\left[45\varphi(1)-154\varphi((N-1)h)+214\varphi((N-2)h)-156\varphi((N-3)h)+\right.$$
$$\left.+61\varphi((N-4)h)-10\varphi((N-5)h)\right].$$

We numerically solve the following integral equations using the optimal quadrature formula constructed in $L_2^{(4)}(0,1)$ space.

Example 1.

$$u(x) = e^x + 2\int_0^1 e^{x+t}u(t)dt.$$

We numerically analyze the given integral equation with the help of the derivative optimal quadrature formula and Simpson's $1/3$ formula when $N=6$. For this, we write down the optimal quadrature formula derived from (1) using finite differences as follows

$$\int_0^1 \varphi(x)dx \cong \sum_{\beta=0}^{6} C_0[\beta]\varphi(h\beta) + \frac{h^2}{12}(\varphi'(0)-\varphi'(1)) + \sum_{\beta=0}^{6} C_1[\beta]\varphi''(h\beta) =$$

$$= \varphi(0)\left[\frac{47h}{144} + \frac{15C_1[0]}{4h^2} - \frac{C_1[2]}{12h^2} - \frac{5C_1[5]}{6h^2}\right] +$$

$$+\varphi(\frac{1}{6})\left[\frac{4h}{3} - \frac{77C_1[0]}{6h^2} + \frac{15C_1[1]}{4h^2} + \frac{4C_1[2]}{3h^2} - \frac{C_1[3]}{12h^2} + \frac{61C_1[5]}{12h^2} - \frac{5C_1[6]}{6h^2}\right] +$$

$$+\varphi(\frac{2}{6})\left[\frac{35h}{48}+\frac{107C_1[0]}{6h^2}-\frac{77C_1[1]}{6h^2}-\frac{5C_1[2]}{2h^2}+\frac{4C_1[3]}{3h^2}-\frac{C_1[4]}{12h^2}-\frac{13C_1[5]}{h^2}+\frac{61C_1[6]}{12h^2}\right]+$$
$$+\varphi(\frac{3}{6})\left[\frac{11h}{9}-\frac{13C_1[0]}{h^2}+\frac{107C_1[1]}{6h^2}+\frac{4C_1[2]}{3h^2}-\frac{5C_1[3]}{2h^2}+\frac{4C_1[4]}{3h^2}+\frac{107C_1[5]}{6h^2}-\frac{13C_1[6]}{h^2}\right]+$$
$$+\varphi(\frac{4}{6})\left[\frac{35h}{48}+\frac{61C_1[0]}{12h^2}-\frac{13C_1[1]}{h^2}-\frac{C_1[2]}{12h^2}+\frac{4C_1[3]}{3h^2}-\frac{5C_1[4]}{2h^2}-\frac{77C_1[5]}{6h^2}+\frac{107C_1[6]}{6h^2}\right]+$$
$$+\varphi(\frac{5}{6})\left[\frac{4h}{3}-\frac{5C_1[0]}{6h^2}+\frac{61C_1[1]}{12h^2}-\frac{C_1[3]}{12h^2}+\frac{4C_1[4]}{3h^2}+\frac{15C_1[5]}{4h^2}-\frac{77C_1[6]}{6h^2}\right]+$$
$$+\varphi(1)\left[\frac{47h}{144}-\frac{5C_1[1]}{6h^2}-\frac{C_1[4]}{12h^2}+\frac{16C_1[6]}{4h^2}\right],$$

where $C_1[\beta]$, $\beta=\overline{0,6}$ are given in Theorem 2.

We write the given integral equation in the form below

$$u(x_i)=e^{x_i}+2e^{x_i}\sum_{j=0}^{6}A_je^ju(t_j),\qquad i=\overline{0,6},$$

and we find the unknown $u(x_i)$ by solving the system of linear algebraic equations. The exact solution of the integral equation is $u(x)=\frac{e^x}{2-e^2}$. In **Table 1** gives a numerical analysis of the solution of the integral equation. **Table 1** shows the error of the Simpson formula through **Error(Simpson 1/3)**, and the error of the derivative optimal quadrature formula through **Error(OQF)**.

Table 1.

x_i	**Exact solution**	**Error (Simpson 1/3)**	**Error (OQF)**
0	-0.1855612526	$1.48*10^{-5}$	$1.33*10^{-6}$
1/6	-0.2192147180	$1.75*10^{-5}$	$1.58*10^{-6}$
2/6	-0.2589715897	$2.07*10^{-5}$	$1.86*10^{-6}$
3/6	-0.3059387842	$2.45*10^{-5}$	$2.20*10^{-6}$
4/6	-0.3614239684	$2.90*10^{-5}$	$2.60*10^{-6}$
5/6	-0.4269719685	$3.42*10^{-5}$	$3.07*10^{-6}$
1	-0.5044077809	$4.04*10^{-5}$	$3.63*10^{-6}$

Example 2.

$$u(x)=x^4+1-\frac{14x}{45}+\int_0^1 xt^4u(t)dt.$$

The exact solution of the integral equation is $u(x)=x^4+1$.

Table 2.

x_i	**Exact solution**	**Error (Simpson 1/3)**	**Error (OQF)**
0	1	0	0
1/6	1.000771605	$2.90*10^{-4}$	$1.61*10^{-4}$
2/6	1.012345679	$5.80*10^{-4}$	$3.22*10^{-4}$
3/6	1.062500000	$8.70*10^{-4}$	$4.84*10^{-4}$
4/6	1.197530864	$1.16*10^{-3}$	$6.45*10^{-4}$
5/6	1.482253086	$1.45*10^{-3}$	$8.07*10^{-4}$
1	2	$1.74*10^{-3}$	$9.68*10^{-4}$

From **Table 1.** and **Table 2.**, it can be seen that the error of the optimal quadrature formula is smaller than the error of the Simpson 1/3 formula.

4 Conclusion

In this research work, a derivative optimal quadrature formula is constructed using the values of the function up to the second derivative at the nodal points for the approximate integration of definite integrals and the numerical solution of integral equations. A representation of the error function corresponding to the difference between the quadrature sum and the definite integral is found. The error function $C_1[\beta]$ is a multivariate function with respect to the coefficients. To find the conditional extremum of a multivariable function, we constructed a Lagrange function. By solving the system of equations, we found an analytical representation of the coefficients. We proved that this quadrature formula exactly integrates polynomials of degree $m-1$. We have demonstrated the effectiveness of the proposed quadrature formulation by numerically solving the integral equations.

5 Acknowledgments

The authors are very thankfull to professors Kh.M. Shadimetov and A.R. Hayotov for discussion the results of the present work.

References

[1] Sobolev S.L. 1974. *Introduction to the Theory of Cubature Formulas,* Nauka, Moscow (in Russian).

[2] Sobolev S.L. 2006. *The coefficients of optimal quadrature formulas,* Selected Works of S.L. Sobolev, Springer, – P. 561–566.

[3] Shadimetov Kh.M., Hayotov A.R. 2011. *Optimal quadrature formulas with positive coefficients in $L_2^{(m)}(0,1)$ space,* Journal of Computational Applied Mathematics, – no. 235, – P. 1114–1128.

[4] Micchelli C.A. 1974. *Best quadrature formulas at equally spaced nodes,* Journal of mathematical analysis and applications, – no. 47, – P. 232–249.

[5] Majeed S.J., Omran H.H. 2008. *Numerical methods for solving linear Fredholm-Volterra integral equations,* Journal of Al-Nahrain University – no. 11(3), – P. 131–134.

[6] Golberg M.A. 1979. *Solution Methods for Integral Equations Theory and Applications,* Plenum Press, New York and London,

[7] Hayotov A.R., Milovanovich G.V., Shadimetov Kh.M. 2011. *On an optimal quadrature formula in the sense of Sard* Numerical Algorithms, – no.57(4), – P. 487–510.

[8] Shadimetov Kh.M., Nuraliev F.A., Kuziev Sh.S. 2024. *Optimal quadrature formula of Hermite type in the space of differentiable functions,* International Journal of Analysis and Applications, – no. 22(25), – P. 1–13.

[9] Shadimetov Kh.M. 1985. *Discrete analogue of the operator d^{2m}/dx^{2m} and its construction,* Problems of Computational and Applied Mathematics – no.79, – P. 22–35.

[10] Shadimetov Kh.M., Hayotov A.R., Nuraliev F.A. 2019. *Optimal interpolation formulas with derivative in the space $L_2^{(m)}(0,1)$,* Filomat, – vol.33(17), – P. 5661–5675.

[11] Sanda M 2023 *Iterative Numerical Methods for a Fredholm–Hammerstein Integral Equation with Modified Argument,* Symmetry, – no. 15(1), https://doi.org/10.3390/sym15010066.

[12] Nuraliev F.A., Kuziev Sh.S. 2023. *The coefficients of an optimal quadrature formula in the space of differentiable functions,* Uzbek Mathematical Journal, – no. 67(2), – P. 124–134.

Received August 07, 2024

УДК 519.644

ПРИБЛИЖЕННОЕ РЕШЕНИЕ ИНТЕГРАЛЬНОГО УРАВНЕНИЯ ФРЕДГОЛЬМА ВТОРОГО РОДА МЕТОДОМ ОПТИМАЛЬНЫХ КВАДРАТУР

[1,2*]***Нуралиев Ф.А.***, [2,3]***Кузиев Ш.С.***, [4]***Джураева К.А.***
[*]f.nuraliev@mail.ru
[1]Ташкентский международный университет,
100084, Узбекистан, Ташкент, ул. Кичик халка йули, 7;
[2]Институт математики им. В.И. Романовского АН РУз,
100174, Узбекистан, г. Ташкент, ул. Университетская, д. 9;
[3]Ферганский политехнический институт,
150100, Узбекистан, г. Фергана, ул. Фергана, 86;
[4]Университет точных и социальных наук,
100069, Узбекистан, Ташкент, улица Карасарой, 341а.

В мире особое внимание уделяется созданию различных оптимальных методов расчета. Статья посвящена построению производных оптимальных квадратурных формул в пространстве дифференцируемых функций с использованием метода Соболева. Эта квадратурная формула состоит из линейной комбинации значений интервала [0,1] до второй производной функции во всех узлах. Погрешность квадратурных формул оценивается нормой функции ошибок. Оптимальную квадратурную формулу получим минимизацией нормы функции ошибок по коэффициентам полученной квадратурной формулы. Полученные оптимальные квадратурные формулы точны для всех степенных функций. Кроме того, приведены некоторые методы численного решения интегральных уравнений Фредгольма второго типа. Этими методами являются производные оптимальные квадратурные формулы и метод Симпсона 1/3. Приводятся числовые примеры, демонстрирующие эффективность и точность представленной работы.

Ключевые слова: пространство Соболева, интегральные уравнения Фредгольма, производная оптимальная квадратурная формула, функционал ошибки, метод Соболева, оптимальные коэффициенты.

Цитирование: *Нуралиев Ф.А., Кузиев Ш.С., Джураева К.А.* Приближенное решение интегрального уравнения Фредгольма второго рода методом оптимальных квадратур // Проблемы вычислительной и прикладной математики. – 2024. – № 4/2(60). – С. 66-73.

ПРОБЛЕМЫ ВЫЧИСЛИТЕЛЬНОЙ И ПРИКЛАДНОЙ МАТЕМАТИКИ

№ 4/2(60) 2024

Журнал основан в 2015 году.
Издается 6 раз в год.

Учредитель:
Научно-исследовательский институт развития цифровых технологий и искусственного интеллекта.

Журнал зарегистрирован в Агентстве информации и массовых коммуникаций при Администрации Президента Республики Узбекистан.
Регистрационное свидетельство №0856 от 5 августа 2015 года.

ISSN 2181-8460, eISSN 2181-046X

Адрес редакции:
100125, г. Ташкент, м-в. Буз-2, 17А.
Тел.: +(998) 712-319-253, 712-319-249.
Э-почта: `journals@airi.uz`.
Веб-сайт: `https://journals.airi.uz`.

Дизайн и вёрстка:
Шарипов Х.Д.

Отпечатано в типографии НИИ РЦТИИ.
Подписано в печать 09.09.2024 г.
Формат 60x84 1/8. Заказ №6. Тираж 100 экз.

PROBLEMS OF COMPUTATIONAL AND APPLIED MATHEMATICS

No. 4/2(60) 2024

The journal was established in 2015.
6 issues are published per year.

Founder:
Digital Technologies and Artificial Intelligence Development Research Institute.

The journal is registered by Agency of Information and Mass Communications under the Administration of the President of the Republic of Uzbekistan.
The registration certificate No. 0856 of 5 August 2015.

ISSN 2181-8460, eISSN 2181-046X

Address:
100125, Tashkent, Buz-2, 17A.
Tel.: +(998) 712-319-253, 712-319-249.
E-mail: journals@airi.uz.
Web-site: https://journals.airi.uz.

Layout design:
Sharipov Kh.D.

DTAIDRI printing office.
Signed for print 09.09.2024
Format 60x84 1/8. Order No. 6. Printed copies 100.

Содержание

Contents