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CONSTRUCTION OF A CUBATURE FORMULA OF THE FIFTH DEGREE OF ACCURACY CONTAINING THE VALUES OF PARTIAL DERIVATIVES

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A number of books by I.P. Mysovkikh, A.H. Stroud, V.I. Krylov and H.T. Shulgina are devoted to the approximate calculation of integrals, where the theory of constructing quadrature and cubature formulas is presented and exact formulas for algebraic polynomials of degree S for standard domains such as n – dimensional cube, n – dimensional ball, surface of n – dimensional ball (sphere), n – dimensional space with weight functions, where are given. In the works by Y.Xu, G.P. Ismatullayev, S.A. Bakhramov, cubature formulas were constructed using orthogonal polynomials by the reproducing kernel method and the Radon method with the minimum number of nodes for the domain and the weight where without the participation of partial derivatives. In this paper, we construct a cubature formula for the domain Ω and the weight exact for polynomials of the fifth degree involving the values partial derivatives at the point $(0, 0)$.

Keywords: cubature formula, degree of algebraic accuracy.

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1 Introduction

The works [7–22] consider the construction of cubature formulas using orthogonal polynomials of the integration domain, as well as formulas with a minimum number of nodes. And in works [23, 24], optimal quadrature formulas are constructed in the Hilbert space.

The method of undetermined parameters for constructing cubature formulas is that the cubature formula is sought in a form that leads to a simplification of systems of nonlinear equations. This turns out to be possible when Ω and $p(x)$ have some symmetry. The simplification is achieved due to the fact that the location of the nodes (and their number) is consistent with the symmetry of Ω and $p(x)$. For the cubature formulas under consideration, both the coefficients and the coordinates of the nodes are unknown.

Here we consider the problem of constructing a cubature formula of the fifth degree of accuracy of the following form

$$\int_0^\infty \int_0^\infty e^{-x-y} x^\alpha y^\alpha f(x, y) dx dy \cong C_{00} f(0, 0)+$$

$$+C_{10}(\frac{\partial f(0, 0)}{\partial x} + \frac{\partial f(0, 0)}{\partial y}) + Af(a, a)+$$

$$+Bf(b,b)+C[f(c,d)+f(d,c)]+D[f(l,m)+f(m,l)], \tag{1}$$

where $\alpha > -1$.

When writing cubature formula (1), it is assumed that the integrand $f(x,y)$ has continuous derivatives. In addition, the symmetry of the region, the weight function $\rho(x,y) = e^{-x-y}x^\alpha y^\alpha$ and the location of the nodes relative to the straight line $y = x$ were taken into account.

For cubature formula (1) to have an algebraic degree of accuracy $S \leqslant 5$, it is sufficient to require its accuracy for monomials

$$1,\ x,\ x^2,\ xy,\ x^3,\ x^2y,\ x^4,\ x^3y,\ x^2y^2\ , x^5,\ x^4y,\ x^3y^2,$$

which is a consequence of the symmetry of the region of integration, the weight function and the location of the nodes relative to the straight line $y = x$. So we get twelve equations as many as there are unknown parameters in the cubature formula (1). From the requirement that the cubature formula (1) be exact to the monomials y, y^2, xy^2, y^3, xy^3, y^4, xy^4, x^2y^3, y^5 we do not obtain new equations, which is a consequence of symmetry.

If this system of twelve nonlinear equations has different real solutions, then the cubature formula (1) will exist.

We require the accuracy of the cubature formula (1) for the following monomials

$$x^2,\ xy,\ x^3,\ x^2y,\ x^4,\ x^3y,\ x^2y^2\ , x^5,\ x^4y,\ x^3y^2,$$

then we have

$$Aa^2+Bb^2+C(c^2+d^2)+D(l^2+m^2)=\mu_{20},$$

$$Aa^2+Bb^2+C2cd+D2lm=\mu_{11},$$

$$Aa^3+Bb^3+C(c^3+d^3)+D(l^3+m^3)=\mu_{30}$$

$$Aa^3+Bb^3+C(c^2d+cd^2)+D(l^2m+lm^2)=\mu_{21},$$

$$Aa^4+Bb^4+C(c^4+d^4)+D(l^4+m^4)=\mu_{40},$$

$$Aa^4+Bb^4+C(c^3d+cd^3)+D(l^3m+lm^3)=\mu_{31},$$

$$Aa^4+Bb^4+C(c^2d^2+d^2c^2)+D(l^2m^2+m^2l^2)=\mu_{22},$$

$$Aa^5+Bb^5+C(c^5+d^5)+D(l^5+m^5)=\mu_{50},$$

$$Aa^5+Bb^5+C(c^4d+cd^4)+D(l^4m+lm^4=\mu_{41},$$

$$Aa^5+Bb^5+C(c^3d^2+c^2d^3)+D(l^3m^2+l^2m^3)=\mu_{32}.$$

To solve this system, we will proceed as follows. We subtract the following equation of the same order from the previous equation, then we obtain

$$\begin{cases} C(c-d)^2 + D(l-m)^2 = \mu_{20} - \mu_{11}, \\ C(c-d)^2(c+d) + D(l-m)^2(l+m) = \mu_{30} - \mu_{21}, \\ C(c-d)^2(c^2+cd+d^2)+ \\ +D(l-m)^2(l^2+lm+m^2) = \mu_{40} - \mu_{31}, \\ C(c-d)^2cd + D(l-m)^2lm = \mu_{31} - \mu_{22}, \\ C(c-d)^2(c+d)(c^2+d^2)+= \\ +D(l-m)^2(l+m)(l^2+m^2) = \mu_{50} - \mu_{41}, \\ C(c-d)^2(c+d)cd + D(l-m)^2(l+m)lm = \mu_{41} - \mu_{32}. \end{cases} \tag{2}$$

which is equivalent to the fact that cubature formula (1) is exact for the polynomials

$$x^2 - xy,\ x^3 - x^2y,\ x^4 - x^3y,\ x^3y - x^2y^2,\ x^5 - x^4y,\ x^4y - x^3y^2.$$

In (2), transforming equations of the same order, we obtain the following systems of equations.

$$\begin{cases} C(c-d)^2 + D(l-m)^2 = \mu_{20} - \mu_{11}, \\ C(c-d)^2(c+d) + D(l-m)^2(l+m) = \mu_{30} - \mu_{21}, \\ C(c-d)^2(c+d)^2 + D(l-m)^2(l+m)^2 = \mu_{40} - \mu_{22}, \\ C(c-d)^2(c+d)^3 + D(l-m)^2(l+m)^3 = \\ = \mu_{50} - 2\mu_{32} + \mu_{41}, \end{cases} \tag{3}$$

$$\begin{cases} C(c-d)^2cd + D(l-m)^2lm = \mu_{31} - \mu_{22}, \\ C(c-d)^2(c+d)cd + D(l-m)^2(l+m)lm = \\ = \mu_{41} - \mu_{32}. \end{cases} \tag{4}$$

Let us introduce the notation

$$M = C(c-d)^2, \quad K = D(l-m)^2, \tag{5}$$

and from (3) and (4) we get

$$\begin{cases} M + K = \mu_{20} - \mu_{11}, \\ M(c+d) + K(l+m) = \mu_{30} - \mu_{21}, \\ M(c+d)^2 + K(l+m)^2 = \mu_{40} - \mu_{22}, \\ M(c+d)^3 + K(l+m)^3 = \mu_{50} + \mu_{41} - 2\mu_{32}. \end{cases} \tag{6}$$

$$\begin{cases} Mcd + Klm = \mu_{31} - \mu_{22}, \\ M(c+d)cd + K(l+m)lm = \mu_{41} - \mu_{32}. \end{cases} \tag{7}$$

where

$$\mu_{ij} = \int_0^\infty \int_0^\infty e^{-x-y} x^\alpha y^\alpha x^i y^j dxdy = \Gamma(i+\alpha+1)\Gamma(j+\alpha+1),$$

where i, j are non-negative integers, $\alpha > -1$.

After calculating the right-hand sides of systems (6) and (7), we obtain

$$\begin{cases} M + K = (\alpha+1)\Gamma^2(\alpha+1), \\ M(c+d) + K(l+m) = 2(\alpha+1)(\alpha+2)\Gamma^2(\alpha+1), \\ M(c+d)^2 + K(l+m)^2 = \\ = 2(\alpha+1)(\alpha+2)(2\alpha+5)\Gamma^2(\alpha+1), \\ M(c+d)^3 + K(l+m)^3 = \\ = 4(\alpha+1)(\alpha+2)(\alpha+3)(2\alpha+5)\Gamma^2(\alpha+1). \end{cases} \tag{8}$$

$$\begin{cases} Mcd + Klm = (\alpha+1)^2(\alpha+2)\Gamma^2(\alpha+1), \\ M(c+d)cd + K(l+m)lm = \\ = 2(\alpha+1)^2(\alpha+2)(\alpha+3)\Gamma^2(\alpha+1). \end{cases} \tag{9}$$

System (8) is solved by the Prony method [6]: that is, suppose that $(c+d)$ and $(l+m)$ are the roots of the quadratic equation

$$t^2 + \beta t + \gamma = 0,$$

whose coefficients are determined from the linear system:

$$\begin{cases} \gamma + 2(\alpha+2)\beta + 2(\alpha+2)(2\alpha+5) = 0 \\ \gamma + (2\alpha+5)\beta + 2(\alpha+3)(2\alpha+5) = 0, \end{cases}$$

$$\beta = -2(2\alpha+5),\ \gamma = (2\alpha+4)(2\alpha+5).$$

$$t^2 - 2(2\alpha+5)t + (2\alpha+4)(2\alpha+5) = 0,$$

$$t_1 = 2\alpha+5+\sqrt{2\alpha+5},\ t_2 = 2\alpha+5-\sqrt{2\alpha+5}.$$

We put

$$c + d = 2\alpha+5+\sqrt{2\alpha+5}, \qquad l + m = 2\alpha+5-\sqrt{2\alpha+5}$$

From the first two equations of (8), we obtain

$$M = \frac{(\alpha+1)(2\alpha+5-\sqrt{2\alpha+5})\Gamma^2(\alpha+1)}{4\alpha+10},$$

$$K = \frac{(\alpha+1)(2\alpha+5+\sqrt{2\alpha+5})\Gamma^2(\alpha+1)}{4\alpha+10}.$$

System (9) is linear with respect to cd and lm, since $(c+d)$ and $(l+m)$ are defined, solving system (9) we obtain

$$cd = (\alpha+1)(\alpha+3+\sqrt{2\alpha+5}),$$

$$lm = (\alpha+1)(\alpha+3-\sqrt{2\alpha+5}).$$

Now solving the following systems of equations

$$\begin{cases} c+d = 2\alpha+5+\sqrt{2\alpha+5}, \\ cd = (\alpha+1)(\alpha+3+\sqrt{2\alpha+5}), \end{cases}$$

$$\begin{cases} l+m = 2\alpha+5-\sqrt{2\alpha+5}, \\ lm = (\alpha+1)(\alpha+3-\sqrt{2\alpha+5}), \end{cases}$$

and assuming for definiteness that $c > d$, $l > m$, we obtain

$$c = \frac{2\alpha+5+\sqrt{2\alpha+5}+\sqrt{6(\alpha+3+\sqrt{2\alpha+5})}}{2},$$

$$d = \frac{2\alpha+5+\sqrt{2\alpha+5}-\sqrt{6(\alpha+3+\sqrt{2\alpha+5})}}{2},$$

$$l = \frac{2\alpha+5-\sqrt{2\alpha+5}+\sqrt{6(\alpha+3-\sqrt{2\alpha+5})}}{2},$$

$$m = \frac{2\alpha+5-\sqrt{2\alpha+5}-\sqrt{6(\alpha+3-\sqrt{2\alpha+5})}}{2}.$$

Knowing the values of c, d, l, m from (5) we obtain

$$C = \frac{(\alpha+1)[(\alpha+4)(2\alpha+5)-(3\alpha+8)\sqrt{2\alpha+5}]\Gamma^2(\alpha+1)}{6(4\alpha+10)(\alpha+2)^2},$$

$$D = \frac{(\alpha+1)[(\alpha+4)(2\alpha+5)+(3\alpha+8)\sqrt{2\alpha+5}]\Gamma^2(\alpha+1)}{6(4\alpha+10)(\alpha+2)^2}.$$

To determine the parameters A, B, a, b, we assume that the cubature formula (1) is exact for the polynomials

$$x^2+xy,\; x^3+3x^2y,\; x^4+4x^3y+3x^2y^2,\; x^5+5x^4y+10x^3y^2$$

this is equivalent to the fact that cubature formula (1) is exact for monomials $x^2, xy, x^3, x^2y, x^4, x^3y, x^2y^2, x^5, x^4y, x^3y^2$ so we get

$$\begin{cases} 2[Aa^2+Bb^2]+C(c+d)^2+D(l+m)^2 = \mu_{20}+\mu_{11}, \\ \\ 4[Aa^3+Bb^3]+C(c+d)^3+D(l+m)^3 = \mu_{30}+3\mu_{21}, \\ \\ 8[Aa^4+Bb^4]+C(c+d)^4+D(l+m)^4 = \\ \qquad = \mu_{40}+4\mu_{31}+3\mu_{22}, \\ \\ 16[Aa^5+Bb^5]+C(c+d)^5+D(l+m)^5 = \\ \qquad = \mu_{50}+5\mu_{41}+10\mu_{32}. \end{cases} \tag{10}$$

Now, evaluating the values of the expressions

$$C(c+d)^{2+i}+D(l+m)^{2+i}, \quad i=0,1,2,3,$$

and transferring them to the right-hand side of system (10), we obtain

$$\begin{cases} Aa^2+Bb^2=\frac{2}{3}(\alpha+1)^2\Gamma^2(\alpha+1), \\ \\ Aa^3+Bb^3=\frac{2}{3}(\alpha+1)^2(\alpha+2)\Gamma^2(\alpha+1), \\ \\ Aa^4+Bb^4=\frac{(\alpha+1)^2(\alpha+2)(2\alpha+5)}{3}\Gamma^2(\alpha+1), \\ \\ Aa^5+Bb^5=\frac{1}{3}(\alpha+1)(\alpha+2)(\alpha+3)(2\alpha+5)\Gamma^2(\alpha+1). \end{cases} \tag{11}$$

System (11) is solved by the Prony method, so we present the calculation results:

$$a=\frac{2\alpha+5+\sqrt{2\alpha+5}}{2}, \quad b=\frac{2\alpha+5-\sqrt{2\alpha+5}}{2},$$

$$A=\frac{2(\alpha+1)^2[(\alpha+4)(2\alpha+5)-(3\alpha+8)\sqrt{2\alpha+5}]\Gamma^2(\alpha+1)}{3(2\alpha+5)^2(\alpha+2)^2},$$

$$B=\frac{2(\alpha+1)^2[(\alpha+4)(2\alpha+5)+(3\alpha+8)\sqrt{2\alpha+5}]\Gamma^2(\alpha+1)}{3(2\alpha+5)^2(\alpha+2)^2}.$$

To define C_{00} and C_{10} in (1) we put $f=1,x$ then we get

$$C_{00}=\mu_{00}-A-B-2(C+D),$$

$$C_{10}=\mu_{10}-Aa-Bb-C(c+d)-D(l+m).$$

Having calculated the right-hand sides, we present the found values of the parameters of the cubature formula (1).

Coefficients:

$$C_{00}=\frac{(5\alpha+8)\Gamma^2(\alpha+1)}{(\alpha+2)^2(2\alpha+5)}, \qquad C_{10}=\frac{(\alpha+1)\Gamma^2(\alpha+1)}{(2\alpha+5)(\alpha+2)}.$$

$$A=\frac{2(\alpha+1)^2[(\alpha+4)(2\alpha+5)-(3\alpha+8)\sqrt{2\alpha+5}]\Gamma^2(\alpha+1)}{3(2\alpha+5)^2(\alpha+2)^2},$$

$$B=\frac{2(\alpha+1)^2[(\alpha+4)(2\alpha+5)+(3\alpha+8)\sqrt{2\alpha+5}]\Gamma^2(\alpha+1)}{3(2\alpha+5)^2(\alpha+2)^2}.$$

$$C=\frac{(\alpha+1)[(\alpha+4)(2\alpha+5)-(3\alpha+8)\sqrt{2\alpha+5}]\Gamma^2(\alpha+1)}{6(4\alpha+10)(\alpha+2)^2},$$

$$D=\frac{(\alpha+1)[(\alpha+4)(2\alpha+5)+(3\alpha+8)\sqrt{2\alpha+5}]\Gamma^2(\alpha+1)}{6(4\alpha+10)(\alpha+2)^2}.$$

Nodes:

$(0,0)$, (a,a), (b,b), (c,d), (d,c), (l,m), (m,l), where

$$a = \frac{2\alpha+5+\sqrt{2\alpha+5}}{2},$$

$$b = \frac{2\alpha+5-\sqrt{2\alpha+5}}{2},$$

$$c = \frac{2\alpha+5+\sqrt{2\alpha+5}+\sqrt{6(\alpha+3+\sqrt{2\alpha+5})}}{2},$$

$$d = \frac{2\alpha+5+\sqrt{2\alpha+5}-\sqrt{6(\alpha+3+\sqrt{2\alpha+5})}}{2},$$

$$l = \frac{2\alpha+5-\sqrt{2\alpha+5}+\sqrt{6(\alpha+3-\sqrt{2\alpha+5})}}{2},$$

$$m = \frac{2\alpha+5-\sqrt{2\alpha+5}-\sqrt{6(\alpha+3-\sqrt{2\alpha+5})}}{2}.$$

Thus, all the parameters of the cubature formula (1) are determined, the coefficients are positive, the nodes are real and belong to the region of integration.

2 Conclusion

A cubature formula of the fifth degree of accuracy is constructed with the participation of the values of partial derivatives at the point $(0,0)$.

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ПОСТРОЕНИЕ КУБАТУРНОЙ ФОРМУЛЫ ПЯТОЙ СТЕПЕНИ ТОЧНОСТИ, СОДЕРЖАЩЕЙ ЗНАЧЕНИЯ ЧАСТНЫХ ПРОИЗВОДНЫХ

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Ряд книг И.П. Мысовких, А.Х. Страуд, В.И. Крылов и Х.Т. Шульгиной посвящены приближенному вычислению интегралов, где представлена теория построения квадратурных и кубатурных формул и точные формулы для алгебраических многочленов степени S для стандартных областей, таких как n – мерный куб, n-мерный шар, поверхность n – мерного шара (сферы), n – мерного пространства с весовыми функциями, где заданы. В работах Ю.Сюя, Г.П. Исматуллаева, С.А. Бахрамова, кубатурные формулы построены с использованием ортогональных полиномов методом воспроизводящего ядра и методом Радона с минимальным числом узлов для области и вес где без участия частных производных. В данной работе мы строим кубатурную формулу для области Ω и веса точного для полиномы пятой степени, содержащие значения частных производных в точке $(0, 0)$.

Ключевые слова: кубатурная формула, степень алгебраической точности.

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