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OPTIMAL QUADRATURE FORMULA FOR APPROXIMATE CALCULATION OF SINGULAR INTEGRALS WITH HILBERT KERNEL

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Many mathematical models in science and technology have been reduced to the approximate calculation of singular integrals with Cauchy and Hilbert kernels and the approximate solution of integral equations. In this paper, an optimal quadrature formula for approximate calculation of singular integrals with a Hilbert kernel in the Sobolev space is constructed and analytical formulas for coefficients that minimize its error are obtained from among the coefficients of the quadrature formula. For this, the discrete analogue of the second-order differential operator is used. The resulting quadrature formula is exact to a first-order polynomial.

Keywords: singular integral, Hilbert kernel, error functional, optimal quadrature formula, extremal function, differential operator, optimal coefficients.

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1 Introduction

Singular integrals of various types are widely used in numerous fields of physics and technology: in the theory of elasticity, electrodynamics, aerodynamics, and nuclear physics.

The most widely used are singular integrals with Cauchy and Hilbert kernels

$$\frac{1}{\pi i}\int\limits_{\gamma}\frac{\varphi(\tau)}{\tau-t}d\tau, \tag{1}$$

$$\frac{1}{2\pi}\int\limits_{0}^{2\pi}\varphi(\sigma)\cot\frac{\sigma-s}{2}d\sigma. \tag{2}$$

A number of papers are devoted to the construction and study of various types of quadrature formulas deliberate for calculating singular integrals with the Hilbert kernel (2) on a set of analytic periodic functions. Among these papers, it should be noted [5], [6].

In this paper, we construct an optimal quadrature formula in the Sobolev space $L_2^{(2)}(0,1)$ based on the functional approach. We consider quadrature formulas of the following form

$$\int\limits_{0}^{1}\varphi(x)\cot\pi(x-t)dx\cong\sum_{\beta=0}^{N}C[\beta]\varphi(x_\beta), \tag{3}$$

where $0 < t < 1$, $\varphi(x)$ is an integrand function, $\varphi(x) \in L_2^{(2)}(0,1)$, $C[\beta]$ are coefficients, $x_\beta = h\beta$ are nodes, $h\beta = [\beta]$, $h = \frac{1}{N}$ and $N \in \mathbb{N}$.

The functions $\varphi(x)$ in $L_2^{(2)}(0,1)$ which is defined as:

$$L_2^{(2)}(0,1) = \{\varphi : [0,1] \to \mathbb{R}, \varphi'(x) \text{ is absolutely continuous}, \varphi''(x) \in L_2(0,1)\},$$

The speace $L_2^{(2)}(0,1)$ is a Hilbert speace with the inner product

$$< \varphi, \psi >= \int_0^1 \varphi''(x) \cdot \psi''(x) dx \tag{4}$$

and in this space the norm corresponding to the inner product (2) is defined as follows

$$\|\varphi\| = \left(\int_0^1 \left(\varphi''(x)\right)^2 dx \right)^{\frac{1}{2}}. \tag{5}$$

The difference between the integral and the quadrature sum

$$(\ell, \varphi) = \int_{-\infty}^{\infty} \ell(x)\varphi(x)dx = \int_0^1 \varphi(x) \cot \pi(x-t)dx - \sum_{\beta=0}^{N} C[\beta]\varphi(x_\beta), \tag{6}$$

is called *the error* of the quadrature formula (1) and gives the value of the error functional ℓ at a function φ. The error functional ℓ has the following representation:

$$\ell(x) = \varepsilon_{[0,1]}(x) \cot \pi(x-t) - \sum_{\beta=0}^{N} C[\beta]\delta(x - x_\beta), \tag{7}$$

here $\varepsilon_{[0,1]}(x)$ is the characteristic function of the interval $[0,1]$, and $\delta(x)$ is the Dirac delta-function.

Since the error functional ℓ is defined on the space $L_2^{(2)}(0,1)$, the following conditions should be imposed

$$(\ell, 1) = 0, \tag{8}$$

$$(\ell, x) = 0. \tag{9}$$

Equalities (6) and (7) show that quadrature formula (1) is exact to the function $d_1 x + d_0$.

It is clear that the coefficients $C[\beta]$ are variable parameters of the quadrature formula (1). A quadrature formula of the form (1) which has the error functional with the minimum norm by coefficients $C[\beta]$ for a given number N of the nodes is called *the optimal quadrature formula* in the space $L_2^{(2)}(0,1)$.

By the Cauchy-Schwarz inequality for all $\varphi \in L_2^{(2)}(0,1)$

$$|(\ell, \varphi)| \leqslant \|\ell | L_2^{(2)*}\| \cdot \|\varphi | L_2^{(2)}\|, \tag{10}$$

the error (4) is estimated by the norm of the error functional (5), i.e.,

$$\|\ell | L_2^{(2)*}\| = \sup_{\|\varphi\| \neq 0} \frac{|(\ell, \varphi)|}{\|\varphi | L_2^{(2)}\|}.$$

In this way, the error estimate of the quadrature formula (1) on the space $L_2^{(2)}(0,1)$ can be reduced to finding a norm of the error functional ℓ in the conjugate space $L_2^{(2)^*}(0,1)$.

There are several methods of construction of optimal quadrature formulas in the sense of Sard (see e.g. [1]). In the space , $L_2^{(m)}(0,1)$ based on these methods, Sard's problem was investigated by many authors (see, for example, [7]- [12] and references therein).

In this paper we give the solution of Sard's problem in the space $L_2^{(2)}(0,1)$, using Sobolev's method for an arbitrary number of nodes $N+1$. Namely, we find the coefficients $C[\beta]$ (and the error functional $\overset{\circ}{\ell}$) such that

$$\| \overset{\circ}{\ell} |L_2^{(2)*}\| = \inf_{C[\beta]} \|\ell|L_2^{(2)^*}\| \tag{11}$$

Therefore, in order to construct an optimal quadrature formula in the sense of Sard in $L_2^{(2)}(0,1)$, we need to solve the following problems:

Problem 1. *Find the norm of the error functional ℓ for the specified quadrature formula* (1).

Problem 2. *Find the optimal coefficients $\overset{\circ}{C}[\beta]$ which give the minimum for the norm of the error functional* (5).

Furthermore, in the present work we solve problems 1 and 2.

2 The norm of the error functional for the quadrature formula

In this section, we solve Problem 1. To do this, first, we find an extremal function ψ_ℓ for the error functional (5) that satisfies the following equality (see [1, 2])

$$(\ell,\psi_\ell) = \|\ell|L_2^{(2)*}\| \cdot \|\psi_\ell|L_2^{(2)}\|,$$

Since the space $L_2^{(2)}(0,1)$ is a Hilbert space, using Riesz's theorem on the general form of a linear continuous functional, we obtain the following

$$(\ell,\varphi) =<\varphi,\psi_\ell>_{L_2^{(2)}(0,1)}, \tag{12}$$

then we get

$$(\ell,\psi_\ell) = \|\ell\|^2_{L_2^{(2)*}(0,1)}, \tag{13}$$

where

$$\psi_\ell = \ell(x) * G_2(x) + P_1(x), \tag{14}$$

$$G_2(x) = \frac{|x|^3}{12}, \tag{15}$$

$P_1(x)$ is a first degree polynomial, $\psi_\ell(x)$ is the extremal function of quadrature formulas (1) in the space $L_2^{(2)}(0,1)$ (see [1, 2]).

Recall that $*$ is the convolution and the convolution of two functions is defined by the formula

$$\varphi(x) * \psi(x) = \int_{-\infty}^{\infty} \varphi(x-y)\psi(y)dy = \int_{-\infty}^{\infty} \varphi(y)\psi(x-y)dy.$$

Now, using equality (11) we have

$$\|\ell|L_2^{(2)*}(0,1)\|^2 = (\ell,\psi_\ell) = \int\limits_{-\infty}^{\infty} \ell(x)\psi_\ell(x)dx$$

and considering (5) and (12), we obtain the following analytic form for the norm of the error functional ℓ

$$\begin{aligned}\|\ell\|^2 \;&=\; \sum_{\gamma=0}^{N}\sum_{\beta=0}^{N} C[\gamma]C[\beta]\frac{|h\beta-h\gamma|^3}{12} - 2\sum_{\beta=0}^{N} C[\beta]\int\limits_0^1 \frac{|x-h\beta|^3}{12}\cot\pi(x-t)dx + \\ &+\int\limits_0^1\int\limits_0^1 \frac{|x-y|^3}{12}\cot\pi(x-t)\cot\pi(y-t)dxdy. \end{aligned} \tag{16}$$

It can be easily showed that $\|\ell\|^2$ is a real non-negative quantity [9].

Thus, Problem 1 is solved.

Using (14), we find the optimal coefficients of the quadrature formula (1) in the next section.

3 Optimal coefficients of the quadrature formula

In (14), the norm of the error functional ℓ is multidimensional function with respect to the coefficients $C[\beta](\beta=\overline{0,N})$. For finding the point of conditional minimum of the expression (14) under the conditions (6) and (7) we apply the method of undetermined multipliers of Lagrange.

Consider the function

$$\Phi(C[\beta],\lambda) = \|\ell|L_2^{(2)*}(0,1)\|^2 - 2\sum_{\alpha=0}^{1}\lambda_\alpha(\ell,x^\alpha), \tag{17}$$

here $\lambda_\alpha(\alpha=0,1)$ are constants.

From the derived expression (15), $C[\beta](\beta=\overline{0,N})$ in terms of coefficients and λ_0, λ_1 Lagrangian function by taking the partial derivatives and setting them to zero we form the necessary condition for the extremum:

$$\begin{cases} \frac{\partial\Phi}{\partial C[\beta]} = -2\int\limits_0^1 \frac{|x-h\beta|^3}{12}\cot\pi(x-t)dx + 2\sum\limits_{\gamma=0}^{N} C[\gamma]\frac{|h\beta-h\gamma|^3}{12} + 2\lambda_0 + 2\lambda_1(h\beta) = 0, \\ \frac{\partial\Phi}{\partial\lambda_0} = -2\left(\int\limits_0^1 \cot\pi(x-t)dx - \sum\limits_{\beta=0}^{N} C[\beta]\right) = 0, \qquad \beta=0,1,...,N, \\ \frac{\partial\Phi}{\partial\lambda_1} = -2\left(\int\limits_0^1 x\cdot\cot\pi(x-t)dx - \sum\limits_{\beta=0}^{N} C[\beta](h\beta)\right) = 0. \end{cases}$$

Hence we get

$$\sum_{\gamma=0}^{N} C[\gamma]\frac{|h\beta-h\gamma|^3}{12} + \lambda_0 + \lambda_1(h\beta) = f_2[\beta], \qquad \beta=\overline{0,N}, \tag{18}$$

$$\sum_{\beta=0}^{N} C[\beta]) = g_0, \tag{19}$$

$$\sum_{\beta=0}^{N} C[\beta](h\beta) = g_1 \tag{20}$$

where,

$$f_2[\beta] = \int_0^1 \frac{|x-h\beta|^3}{12} \cot \pi(x-t)dx,$$

$$g_0 = \int_0^1 \cot \pi(x-t)dx,$$

$$g_1 = \int_0^1 x \cot \pi(x-t)dx.$$

Let's calculate these $f_2[\beta]$, g_0 and g_1

$$f_2[\beta] = \int_0^1 \frac{|x-h\beta|^3}{12} \cot \pi(x-t)dx =$$

$$= \frac{1}{12}\sum_{k=0}^{\infty}(-1)^k \frac{2^{2k}B_{2k}\pi^{2k-1}}{(2k)!}\left[\frac{1}{3+2k}\left((1-t)^{2k+3} - 2(h\beta-t)^{2k+3} - t^{2k+3}\right)+\right.$$
$$+\frac{3(t-h\beta)}{2+2k}\left((1-t)^{2k+2} - 2(h\beta-t)^{2k+2} + t^{2k+2}\right)+$$
$$\left.\frac{3(t-h\beta)^2}{1+2k}\left((1-t)^{2k+1} - 2(h\beta-t)^{2k+1} - t^{2k+1}\right)\right]+$$
$$+\frac{(t-h\beta)^3}{6\pi}\ln\left|\frac{\sin \pi t}{\sin \pi(h\beta-t)}\right|;$$

Here, B_{2k} are Bernoulli numbers and these numbers are constructed using the following recurrence formula

$$B_0 = 1, \quad B_n = \frac{-1}{n+1}\sum_{k=1}^{n}\binom{n+1}{k+1}B_{n-k}, \qquad n \in N$$

and

$$\int x^p \cot x dx = \sum_{k=0}^{\infty}(-1)^k \frac{2^{2k}B_{2k}}{(2k+p)(2k)!}x^{2k+p}, \qquad p \geqslant 1,\ |x| < \pi$$

(see [4], p. 206),

$$g_0 = \int_0^1 \cot \pi(x-t)dx = 0,$$

$$g_1 = \int_0^1 x \cot \pi(x-t)dx = \sum_{k=0}^{\infty}(-1)^k \frac{2^{2k}B_{2k}\pi^{2k-1}}{(2k+1)(2k)!}\left((1-t)^{2k+1} + t^{2k+1}\right),$$

Our aim is to get the exact solution of the system (16)-(18). Now we introduce auxiliary concepts for solving the system (16)-(18).

4 Supporting concepts

Here we mainly use the concept of functions of discrete argument and operations on them, which are given in [1] .

Definition 1. *A function $\varphi(h\beta)$ is called function of discrete argument if it is defined on some set of integer values of β.*

Definition 2. *We define the inner product of two discrete argument functions $\varphi(h\beta)$ and $\psi(h\beta)$ as the following number*

$$[\varphi,\psi]=\sum_{\beta=-\infty}^{\infty}\varphi(h\beta)\cdot\psi(h\beta), \tag{21}$$

if the series on the right hand side of equality (19) converges absolutely.

Definition 3. *We define convolution of two discrete argument functions $\varphi(h\beta)$ and $\psi(h\beta)$ as the inner product*

$$\varphi(h\beta)*\psi(h\beta)=[\varphi(h\gamma),\psi(h\beta-h\gamma)]=\sum_{\gamma=-\infty}^{\infty}\varphi(h\gamma)\cdot\psi(h\beta-h\gamma).$$

Using these concepts, we solve the system (16)-(18).

Suppose $C[\beta]=0$ for $\beta<0$ and $\beta>N$. Then, using the above definitions, we rewrite system (16) in the convolution from

$$C[\beta]*G_2[\beta]+\lambda_0+\lambda_1[\beta]=f_2[\beta], \qquad \beta=\overline{0,N}. \tag{22}$$

Here we introduce the following definitions discrete argument functions

$$\nu[\beta]=C[\beta]*G_2[\beta],$$

$$u[\beta]=\nu[\beta]+\lambda_0+\lambda_1[\beta] \tag{23}$$

In this statement it is necessary to express the coefficients $C[\beta]$ by the function $u[\beta]$. For this we need the operator $D_2[\beta]$ which satisfies the equality

$$hD_2[\beta]*G_2[\beta]=\delta[\beta], \tag{24}$$

where $\delta[\beta]=\begin{cases}1, & \beta=0,\\ 0, & \beta\neq 0,\end{cases}$ i.e., $\delta[\beta]$ is the discrete delta-function.

It should be noted that the discrete analogue $D_2[\beta]$ of the differential operator d^{2m}/dx^{2m} was constructed by Kh.M.Shadimetov [7]. In particular, when $m=2$ from the results of the work [7] we get the following form of the discrete analogue of the differential operator d^4/dx^4 satisfying equality (22)

$$D_2[\beta]=\frac{6}{h^4}\begin{cases}6\sqrt{3}q^{|\beta|}, & |\beta|\geqslant 2,\\ 19-12\sqrt{3}, & |\beta|=1,\\ 6\sqrt{3}-8, & \beta=0,\end{cases} \tag{25}$$

where $q=\sqrt{3}-2$.

Furthermore, we need the following properties of the discrete operator $D_2[\beta]$ which were proved in [7]

$$D_2[\beta] * [\beta]^\alpha = 0, \qquad \alpha = 0, 1. \tag{26}$$

Then taking (21) and (23) into account and using the last equalities we arrive to the following

$$C[\beta] = hD_2[\beta] * u[\beta], \quad \text{where } \beta \in \mathbb{Z}. \tag{27}$$

For calculating this convolution, i.e., to get optimal coefficients $C[\beta]$, the function $u[\beta]$ should be determined for all integer values of β. It is clear from (16) that

$$u[\beta] = f_2[\beta], \quad \text{for } \beta = \overline{0, N}. \tag{28}$$

Now we have to find the representation of $u[\beta]$ for $\beta = -1, -2, ...$ and $\beta = N+1, N+2, ...$ Using (17) and (18), taking (25) into account, from (21) for $u[\beta]$ we obtain

$$u[\beta] = \begin{cases} -\frac{g_0}{12}[\beta]^3 + \frac{g_1}{4}[\beta]^2 + a_1^-[\beta] + a_0^-, & \beta \leqslant 0, \\ f_2[\beta], & 0 \leqslant \beta \leqslant N, \\ \frac{g_0}{12}[\beta]^3 - \frac{g_1}{4}[\beta]^2 + a_1^+[\beta] + a_0^+, & \beta \geqslant N, \end{cases}$$

Since $g_0 = 0$, we rewrite the last expression

$$u[\beta] = \begin{cases} \frac{g_1}{4}[\beta]^2 + a_1^-[\beta] + a_0^-, & \beta \leqslant 0, \\ f_2[\beta], & 0 \leqslant \beta \leqslant N, \\ -\frac{g_1}{4}[\beta]^2 + a_1^+[\beta] + a_0^+, & \beta \geqslant N, \end{cases} \tag{29}$$

where a_1^-, a_1^+, a_0^- and a_0^+ are unknowns and

$$\begin{array}{ll} a_1^- = \lambda_1 - p_1 & a_0^- = \lambda_0 + p_0, \\ a_1^+ = \lambda_1 + p_1, & a_0^+ = \lambda_0 - p_0, \end{array} \tag{30}$$

where

$$p_1 = \frac{1}{4}\sum_{\gamma=0}^{N} C[\gamma](h\gamma)^2, \qquad p_0 = \frac{1}{12}\sum_{\gamma=0}^{N} C[\gamma](h\gamma)^3. \tag{31}$$

Now, it is enough to find unknowns a_1^-, a_1^+, a_0^- and a_0^+. Then optimal coefficients $C[\beta], \beta = 0, 1, ..., N$ are found from (24). For finding a_1^-, a_1^+, a_0^- and a_0^+, we take account our assumption that $C[\beta] = 0$ for $\beta = -1, -2, ...$ and $\beta = N+1, N+2, ...$ and from (24) we get the equation

$$D_2[\beta] * u[\beta] = 0 \text{ for } \beta < 0 \text{ and } \beta > N. \tag{32}$$

First from (26) when $\beta = 0$ and $\beta = N$ we get the following

$$a_0^- = f_0[0], \qquad a_0^+ = f_2[N] + \tfrac{g_1}{4} - a_1^+. \tag{33}$$

Now, taking into account a_0^- and a_0^+, for $u[\beta]$ we have

$$u[\beta] = \begin{cases} \frac{g_1}{4}[\beta]^2 + a_1^-[\beta] + f_2[0], & \beta \leqslant 0, \\ f_2[\beta], & 0 \leqslant \beta \leqslant N, \\ \frac{g_1}{4}(1 - [\beta]^2) + ([\beta] - 1)a_1^+ + f_2[N], & \beta \geqslant N. \end{cases} \tag{34}$$

Then only the unknowns a_1^- and a_1^+ remain in the form of $u[\beta]$. To find them, we solve equation (30) for $\beta = -1$ and $\beta = N+1$. Thus, we have defined the complete representation of $u[\beta]$. Now, using the expression (24), we find the optimal coefficients of the quadrature formula (1).

Theorem 1. *The coefficients of the optimal quadrature formula in the sense of Sard of the form* (1) *in the space* $L_2^{(2)}(0,1)$ *have the following form*

$$C[\beta]=\frac{6}{h^3}\begin{cases}\Big[(q+1)a_1^-h+(3q+2)f_2[0]+f_2[1]+6(q+2)\sum\limits_{\gamma=1}^{N}q^{\gamma}f_2[\gamma]+\\ +q^N\Big(\frac{g_1h^2}{4}-(q+2)(ha_1^+-\frac{hg_1}{2})-3(q+1)f_2[N]\Big)\Big], & \beta=0,\\ \Big[q^{\beta}\Big(-\frac{g_1h^2}{4}+(q+2)a_1^-h-3(q+1)f_2[0]\Big)+6(q+2)\sum\limits_{\gamma=0}^{\beta-2}q^{\beta-\gamma}f_2[\gamma]-\\ -(12q+5)f_2[\beta-1]+(6q+4)f_2[\beta]-(12q+5)f_2[\beta+1]+\\ +6(q+2)\sum\limits_{\gamma=\beta+2}^{N}q^{\gamma-\beta}f_2[\gamma]+q^{N-\beta}\Big(\frac{h^2g_1}{4}-(q+2)(a_1^+h-\frac{hg_1}{2})-\\ -3(q+1)f_2[N]\Big)\Big], \qquad \beta=\overline{1,N-1}, & \\ \Big[\frac{q+1}{2}g_1h-(q+1)a_1^+h-(3q+10)f_2[N]+6(q+2)\sum\limits_{\gamma=0}^{N}q^{N-\gamma}f_2[\gamma]+\\ +f_2[N-1]+q^N\Big(-\frac{g_1h^2}{4}+(q+2)a_1^-h-3(q+1)f_2[0]\Big)\Big], & \beta=N,\end{cases}$$

With this, we also solved Problem 2.

5 Conclusion

In this paper, the optimal coefficients of the optimal quadrature formula were found using the Sobolev method for singular integrals with a Hilbert kernel in $L_2^{(2)}(0,1)$ Sobolev space. In this method, the Lagrange function was constructed to find the conditional minimum of the multivariable function, and the discrete analogue of the differential operator d^4/dx^4 was used to find the optimal coefficients. The expressions of optimal coefficients that minimize this system were found.

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ОПТИМАЛЬНАЯ КВАДРАТУРНАЯ ФОРМУЛА ДЛЯ ПРИБЛИЖЕННОГО ВЫЧИСЛЕНИЯ СИНГУЛЯРНЫХ ИНТЕГРАЛОВ С ЯДРОМ ГИЛЬБЕРТА

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Многие математические модели в науке и технике доведены до приближенного вычисления сингулярных интегралов с ядрами Коши и Гильберта и приближенного решения интегральных уравнений. В данной работе строится оптимальная квадратурная формула для приближенного вычисления сингулярных интегралов с ядром Гильберта в пространстве Соболева, т.е. из числа коэффициентов квадратурной формулы получаются аналитические формулы для коэффициентов, минимизирующих ее погрешность. Для этого используется дискретный аналог дифференциального оператора четвертого порядка по методу С.Л. Соболева, и полученная квадратурная формула имеет точность до полинома первого порядка.

Ключевые слова: сингулярный интеграл с ядром Гильберта, функционал погрешности, оптимальная квадратурная формула, экстремальная функция, дифференциальный оператор, оптимальные коэффициенты.

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