

UDC 519.652.3

HISTOSPLINE FUNCTIONS IN RECONSTRUCTING FUNCTIONS USING INTEGRAL VALUES

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Background: Reconstructing continuous functions from their integral values over specific intervals is a significant problem in applied and computational mathematics, especially when direct node values are unavailable. Several researchers have conducted studies in this area, proposing various methods to reconstruct functions, such as traditional approaches, algebraic techniques and others.

Purpose: The objective of this paper is to study the histospline method in Sobolev spaces to reconstruct continuous functions given by their interval integral values and to obtain explicit representation formulas for the coefficients.

Method: The histospline method is investigated in the Sobolev spaces to reconstruct functions given by their interval integral values. It is shown that when the derivative is applied to the natural spline, the degree of the histospline function decreases by one.

Results: As a result, a piecewise constant function is obtained in the Sobolev space of functions with square-integrable first-order derivatives on the unit interval, while a quadratic function is formed in the Sobolev space of functions with square-integrable second-order derivatives on the unit interval. Explicit formulas for the coefficients of the histopolating spline are derived, and numerical experiments are presented.

Implication: The constructed histosplines ensure an exact match for the area of the respective histogram bars, providing an efficient framework for function reconstruction in low-order Sobolev spaces.

Keywords: approximation, natural spline, histogram, histopolation, histospline

Citation: Hayotov A.R., Mukhtorova M.M. 2026. Histospline functions in reconstructing functions using integral values. *Problems of Computational and Applied Mathematics*. 4/1(75): 5-15.

DOI: 10.71310/pcam.4-1_75.2026.01

1 Introduction

Reconstruction of continuous functions plays an important role in applied mathematics and computational methods. Unlike traditional interpolation and approximation problems, this approach relies on the integral values of the function over specific intervals rather than its values at the nodes. The main goal of this approach is that the integral value of the smooth function over each interval corresponds exactly to the area of the respective histogram bar. Constructing a histospline from a histogram is useful because it provides a smoother representation of the data distribution. While a histogram shows the data in separate intervals, a histospline connects these intervals and gives a continuous curve. This makes it easier to observe the overall pattern of the distribution.

In practical application, histosplines can be used to analyze statistical data in areas such as economics, engineering, medicine and environmental studies. They help researchers better understand the shape of the distribution, identify trends, and estimate

the probability density more accurately. A smoother curve may also reduce the influence of random fluctuations in the data.

Another advantage is that histosplines improve the visual interpretation of large datasets. They provide a clearer picture of how the observations are distributed and can support statistical modeling and forecasting. Because of these features, histosplines are considered a useful tool for data analysis and decision-making in many research fields. There are several researchers investigated new methods about this issue. Many books are devoted to the theory of splines, for example, I. J. Schoenberg [1], L. L. Schumaker [2] and others. Using Sobolev's method, interpolation D^m -splines that minimize the expressions $\int_0^1 (\varphi^{(m)}(x))^2 dx$ in the $L_2^{(m)}(0, 1)$ space are constructed by A. Cabada, A. R. Hayotov, Kh. M. Shadimetov [3]. We construct a new histospline using D^m -spline and its coefficients. The algorithm for constructing the histospline is detailed in the works [4, 5].

In [6], Behforooz first introduced the motivation of integro cubic splines to approximate the function and analyzed the approximation problem with various and conditions. Recently, spline functions and their practical applications have attracted the interest of researchers because of their significance on computational mathematics and applied mathematics. In 2006, new method of constructing cubic splines with integral values of the functions were introduced by Behforooz [7]. In 2015, Wu and Zhang [8] discussed the integro quadratic spline interpolation by using B-splines. In [5], cubic spline histopolation with arbitrary placement of histogram knots and spline knots between them is studied. Histopolating spline is represented with the help of second moments and particular integrals. In [9], the problem of the approximation of a given histogram by a function from Sobolev space under inequality constraints for area matching conditions is considered by N. Budkina. In [10], with the help of Marsden's identity quasi-interpolant schemes that produce the space of algebraic hyperbolic functions are obtained. In [11], new C^1 cubic Hermite interpolation schemes employing algebraic trigonometric splines are introduced. One technique fits values of a function and its derivative directly, whereas the other preserves monotonicity by inserting additional knots into a low-degree spline, with computational examples illustrating their robustness. Barrera, Eddargani and Lamnii introduce an efficient spline quasi-interpolation operator based on subinterval integrals that reproduces constant polynomials and algebraic hyperbolic functions without requiring additional boundary conditions or system resolution [12]. Ajeddar and Lamnii establish refinement and subdivision algorithms for uniform algebraic hyperbolic splines with tension parameters, leading to a new multiresolution framework based on smooth reverse subdivision for general topological curves [13]. In [14], by analyzing integro spline characteristics, the authors present a low-complexity construction algorithm to build splines of degree $m + 1$ from degree m , highlighting that local integro-sextic splines bypass the heavy computational demands associated with non-local schemes. In [15], by utilizing integral values across partition subintervals instead of direct point evaluations, the study proposes a globally C^2 cubic Hermite scheme capable of exact representation for linear and hyperbolic spaces with optimal approximation accuracy. In [16], the paper proposes a framework combining histograms with cubic B-spline curves to accurately estimate probability density functions (PDFs) from vehicle-related time series data. Reference [17] introduces a stable numerical approach combining extended cubic B-spline functions and Caputo derivatives to solve non-linear time-fractional integro-differential equations containing weakly singular kernels.

Zhanlav and Mijiddorj construct integro cubic splines on non-uniform partitions using integral values of an unknown function, establishing a consistency relation to derive

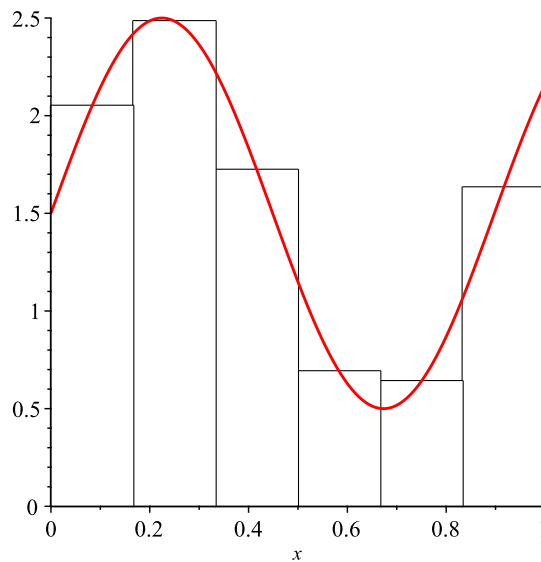
local constructions while analyzing their approximation and convexity properties [18]. In [19], the authors propose quartic and sextic C^3 -spline methods for numerically solving fractional integro-differential equations, providing detailed convergence analysis alongside illustrative numerical examples.

Our paper is organized as follows. In Section 2, we present the algorithm of constructing the histospline in $L_2^{(1)}(0, 1)$ space and the formulas for calculating the coefficients. In Section 3, we expose the method for the construction of the histospline in $L_2^{(2)}(0, 1)$ space and the expressions for coefficients. Section 4 is devoted to numerical experiments demonstrate that by increasing the values m and N the absolute errors between histosplines and the given functions decrease. Eventually, we deduce our paper in Section 5.

2 Problem statement and method

In this work, we consider the problem of optimal approximation of a function in a space $L_2^{(1)}(0, 1)$ with known integral values:

$$f(x) \cong P_f(x). \quad (1)$$



We aim to construct a linear spline using given integral values and then derive a histospline from it using the differentiation operator.

Suppose a function f in (1) belongs to the space $L_2^{(1)}(0, 1)$ of functions with a square-integrable first derivative. At the same time, on the interval $[0, 1]$

$$0 = x_0 < x_1 < \dots < x_n = 1$$

are nodes and on each subinterval, the definite integrals

$$S_i = \int_{x_{i-1}}^{x_i} f(x) dx, \quad i = 1, 2, \dots, n,$$

of the function are known.

According to Newton–Leibniz’s formula, we get $A_0 = S(x_0) = 0$. We find the values at subsequent points by cumulatively summing the integrals over the given intervals. We create the cumulative sum of the histogram areas as follows:

$$\begin{aligned} A_0 &= 0, \\ A_1 &= S_1, \\ A_2 &= S_1 + S_2, \\ &\dots \\ A_n &= S_1 + S_2 + \dots + S_n = \sum_{i=1}^n S_i. \end{aligned}$$

The resulting values of A_n are the exact values of this integral function at the nodes. Using the areas of the given histogram, we construct the following table:

x	x_0	x_1	$\dots$	x_n
A	0	S_1	$\dots$	$\sum_{j=1}^n S_j$

The function $Q_1(x) \in L_2^{(1)}(0, 1)$ which minimizes $\int_0^1 (\varphi'(x))^2 dx$ and meets the interpolation conditions $Q_1(h\beta) = \varphi(h\beta)$, $\beta = 0, 1, \dots, n$, where $h = 1/n$, $n = 1, 2, \dots$, $\varphi(x) \in L_2^{(1)}(0, 1)$, is expressed below:

$$Q_1(x) = \sum_{\gamma=0}^n C_\gamma \frac{|x - x_\gamma|}{2} + p_0. \quad (2)$$

We can find the coefficients C_γ , $\gamma = 0, 1, \dots, N$ and p_0 are determined by system

$$\begin{aligned} \sum_{\gamma=0}^n C_\gamma \frac{|h\beta - h\gamma|}{2} + p_0 &= \varphi(h\beta), \quad \beta = 0, 1, \dots, n, \\ \sum_{\gamma=0}^n C_\gamma &= 0. \end{aligned}$$

The algorithm of constructing a natural spline $Q_1(x)$ and the exact formulas for obtaining its coefficients are described in reference [3].

To find the histospline function, we take the derivative of the spline (2) [9]:

$$f(x) \cong P_f(x) = \frac{d}{dx} Q_1(x).$$

As a result, we obtain the following piecewise constant function:

$$P_1(x) = \frac{d}{dx} Q_1(x) = \sum_{\gamma=0}^n C_\gamma \frac{\text{sgn}(x - x_\gamma)}{2}. \quad (3)$$

When we take the derivative of the previously constructed spline, the degree of the spline decreases by one. As a result, on each interval, the histospline takes the form of a piecewise function. Based on the integral conservation condition, this constant value is exactly equal to the height of the histogram in that interval.

The $P_f(x)$ spline found by derivation preserves the area under each column of the histogram. The reason is that the following equality holds:

$$S_i = \int_{x_{i-1}}^{x_i} f(x) dx = Q(x_i) - Q(x_{i-1}) = A_i - A_{i-1} = \sum_{j=1}^i S_j - \sum_{j=1}^{i-1} S_j = S_i.$$

Theorem 1. *The coefficients of the histospline $P_1(x)$ defined by (3) have the following form:*

$$\begin{aligned} C_0 &= \frac{1}{h}(A_1 - A_0) = \frac{S_1}{h}, \\ C_\beta &= \frac{1}{h}(A_{\beta-1} - 2A_\beta + A_{\beta+1}) = \frac{S_{\beta+1} - S_\beta}{h}, \\ C_n &= \frac{1}{h}(A_{n-1} - A_n) = -\frac{S_n}{h}, \\ p_0 &= \frac{1}{2}(A_0 + A_n) = \frac{1}{2} \sum_{i=1}^n S_i. \end{aligned}$$

3 The histospline in the space $L_2^{(2)}(0, 1)$

The function $Q_2(x) \in L_2^{(2)}(0, 1)$ can minimize $\int_0^1 (\varphi''(x))^2 dx$ and satisfy the interpolation conditions $Q_2(h\beta) = \varphi(h\beta)$, $\beta = 0, 1, \dots, n$, where $h = 1/n$, $n = 1, 2, \dots$, $\varphi(x) \in L_2^{(2)}(0, 1)$.

Here the interpolation spline $Q_2(x)$:

$$Q_2(x) = \sum_{\gamma=0}^n C_\gamma \frac{|x - x_\gamma|^3}{12} + p_1 x + p_0. \quad (4)$$

The coefficients of $Q_2(x)$ are identified by the system

$$\begin{aligned} \sum_{\gamma=0}^n C_\gamma \frac{|h\beta - h\gamma|^3}{12} + p_1 \cdot (h\beta) + p_0 &= \varphi(h\beta), \quad \beta = 0, 1, \dots, n, \\ \sum_{\gamma=0}^n C_\gamma &= 0, \\ \sum_{\gamma=0}^n C_\gamma \cdot (h\gamma) &= 0. \end{aligned}$$

Based on the algorithm in reference [3], we have defined the natural spline $Q_2(x)$ above to construct the histospline function in the space $L_2^{(2)}(0, 1)$.

Taking the derivative of the spline (4), the histospline function $P_2(x)$ is obtained [9]:

$$P_f(x) = \frac{d}{dx} Q_2(x).$$

The degree of the spline $Q_2(x)$ is equal to 3. Taking its derivative, the degree of the histospline decreases by one and becomes equal to 2. As a result, we obtain the following quadratic function:

$$P_2(x) = \frac{d}{dx} Q_2(x) = \sum_{\gamma=0}^n C_\gamma \frac{(x - x_\gamma)^2 \operatorname{sgn}(x - x_\gamma)}{4} + p_1. \quad (5)$$

We need some concepts below to determine coefficients of the histospline. The family of Euler–Frobenius polynomials $E_k(x)$ for $k = 1, 2, \dots$ can be explicitly introduced via the relation [20]:

$$E_k(x) = \frac{(1-x)^{k+2}}{x} \left(x \frac{d}{dx} \right)^k \frac{x}{(1-x)^2},$$

under the initial condition $E_0(x) = 1$.

Theorem 2. *For the equally spaced nodes in the space $L_2^{(2)}(0, 1)$, the coefficients of the above histospline (5) $P_f(x)$ have the following form:*

$$\begin{aligned} C_0 &= \frac{6}{h^3} [A_1 - 7A_0 - p_1^- h] + \frac{36\sqrt{3}}{h^3} \left[\sum_{\gamma=1}^n q_1^\gamma A_\gamma + M_1 + q_1^n N_1 \right], \\ C_\beta &= \frac{6}{h^3} [A_{\beta-1} - 8A_\beta - A_{\beta+1}] + \frac{36\sqrt{3}}{h^3} \left[\sum_{\gamma=1}^n q_1^{|\beta-\gamma|} A_\gamma + q_1^\beta M_1 + q_1^{n-\beta} N_1 \right], \\ \beta &= 1, 2, \dots, n-1, \\ C_n &= \frac{6}{h^3} [A_{n-1} - 7A_n + p_1^+ h] + \frac{36\sqrt{3}}{h^3} \left[\sum_{\gamma=1}^n q_1^{n-\gamma} A_\gamma + q_1^n M_1 + N_1 \right], \\ p_\alpha &= \frac{1}{2}(p_\alpha^+ + p_\alpha^-), \quad \alpha = 0, 1, \end{aligned}$$

where

$$\begin{aligned} M_1 &= -p_1^- \frac{hq_1}{(1-q_1)^2} + p_0^- \frac{q_1}{1-q_1}, \\ N_1 &= -p_1^+ \left(\frac{hq_1}{(1-q_1)^2} + \frac{q_1}{1-q_1} \right) + p_0^+ \frac{q_1}{1-q_1}, \\ p_0^- &= A_0, \quad p_0^+ = A_n - p_1^+, \\ p_1^- &= \frac{T_1 A_{11}^+ - S_1 B_{11}^+}{B_{11}^- A_{11}^+ - A_{11}^- B_{11}^+}, \quad p_1^+ = \frac{S_1 B_{11}^- - T_1 A_{11}^-}{B_{11}^- A_{11}^+ - A_{11}^- B_{11}^+}, \\ B_{11}^- &= h(7q_1 + 2), \quad B_{11}^+ = -h\sqrt{3}q_1^{n+2}, \\ A_{11}^- &= h\sqrt{3}q_1^{n+2}, \quad A_{11}^+ = -h(7q_1 + 2), \\ T_1 &= -6\sqrt{3}q_1^2 \sum_{\gamma=0}^n q_1^\gamma A_\gamma + A_0(2q_1 + 1)(1 - q_1) + A_n\sqrt{3}q_1^{n+2}(1 - q_1), \\ S_1 &= -6\sqrt{3}q_1^2 \sum_{\gamma=1}^n q_1^{n-\gamma} A_\gamma + A_0\sqrt{3}q_1^{n+2}(1 - q_1) + A_n(2q_1 + 1)(1 - q_1), \\ q_1 &= \sqrt{3} - 2, \end{aligned}$$

where q_1 is the characteristic root of the Euler–Frobenius polynomial $E_2(x)$ (satisfying $|q_1| < 1$), and $p_0^-, p_0^+, p_1^-, p_1^+$ are the constants determined by the natural boundary conditions at the endpoints $x = 0$ and $x = 1$.

4 Results and discussion

In this section, we present the numerical results obtained by applying Theorem 1 and Theorem 2. For our calculations, we test two functions: $f_1(x) = e^x$ and $f_2(x) = \sin x$. We denote their corresponding histosplines from (3) and (5) as $S_m(f_1; x)$ and $S_m(f_2; x)$.

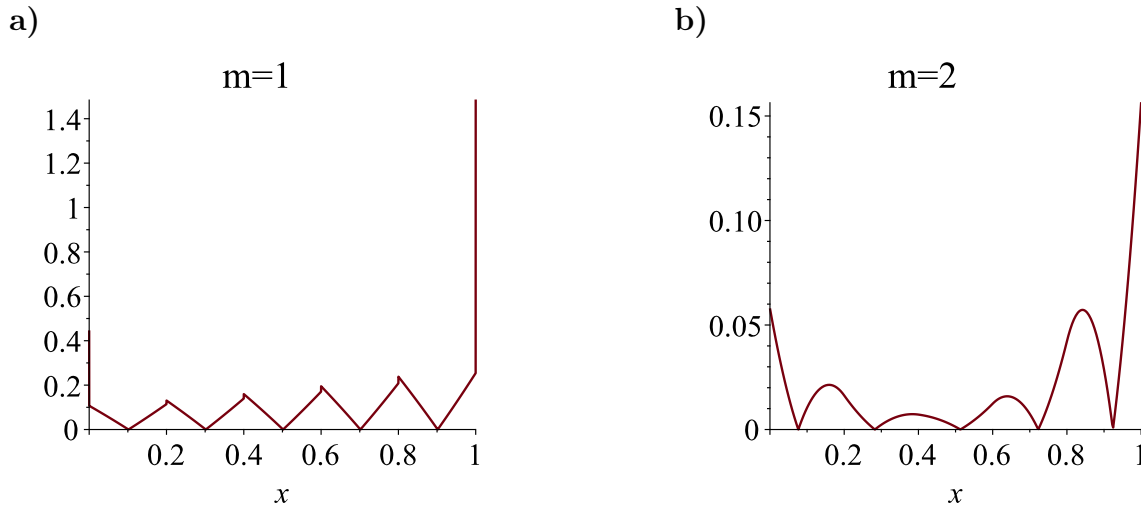


Figure 1 Graphs of absolute errors $|f_1(x) - S_m(f_1; x)|$ for $N = 5$ with $m = 1, 2$.

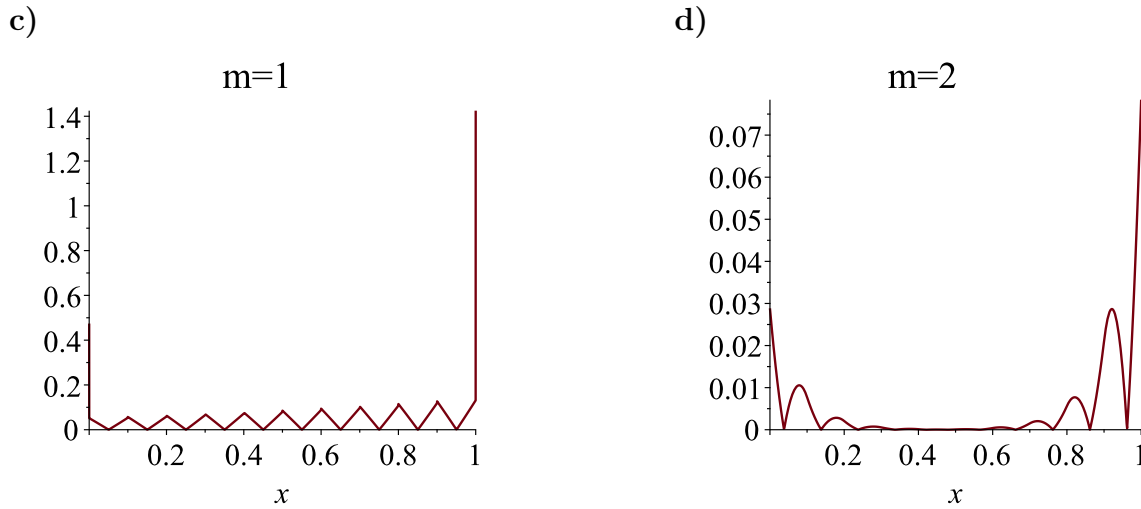


Figure 2 Graphs of absolute errors $|f_1(x) - S_m(f_1; x)|$ for $N = 10$ with $m = 1, 2$.

We calculated the absolute errors using $S_m(x)$ with $N = 5, 10$ and $m = 1, 2$. The plots of these errors are shown in Figures 1–4. Looking at these graphs, it is clear that when we increase the parameters m and N , the absolute errors between the real functions and the splines become smaller.

The error plots show that for $m = 1$ and $m = 2$, the maximum error occurs near the boundaries of the $[0, 1]$ interval. These boundary errors typically depend on the nature of the boundary conditions. We plan to minimize these errors and improve the accuracy of the approximation near the boundaries in future studies.

An analysis of the obtained numerical results and errors is presented in Table 1 and Table 2. The maximum absolute error was calculated for functions $f_1(x) = e^x$ and $f_2(x) = \sin x$ in the $L_2^{(1)}(0, 1)$ and $L_2^{(2)}(0, 1)$ spaces.

In the case of $m = 1$, the error decreased very slowly even as the number of nodes increased from 10 to 100. However, for $m = 2$, the error for function $f_1(x) = e^x$ dropped from 0.07840 to 0.00784, a reduction of more than 10. For the periodic function $f_2(x) =$

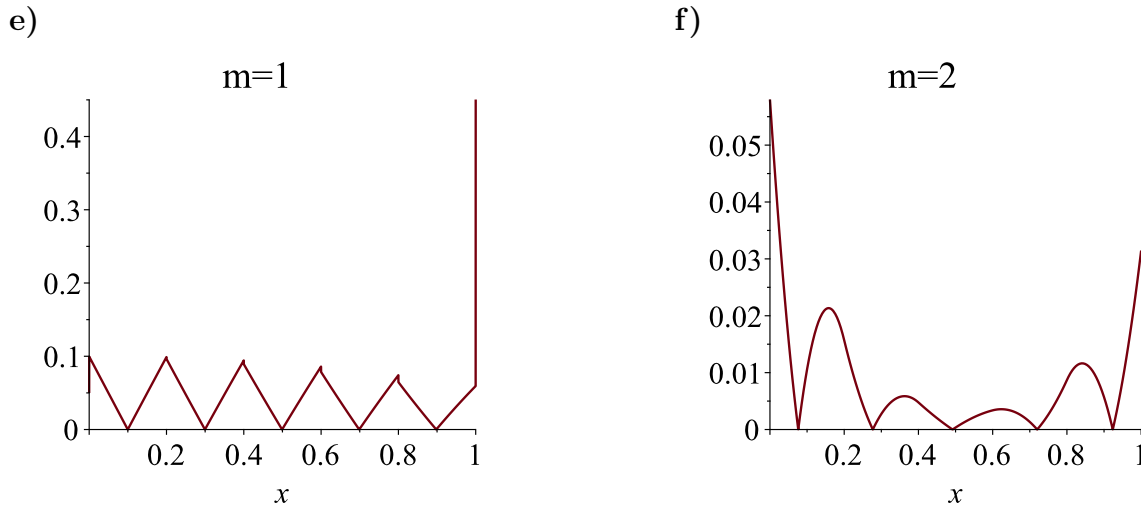


Figure 3 Graphs of absolute errors $|f_2(x) - S_m(f_2; x)|$ for $N = 5$ with $m = 1, 2$.

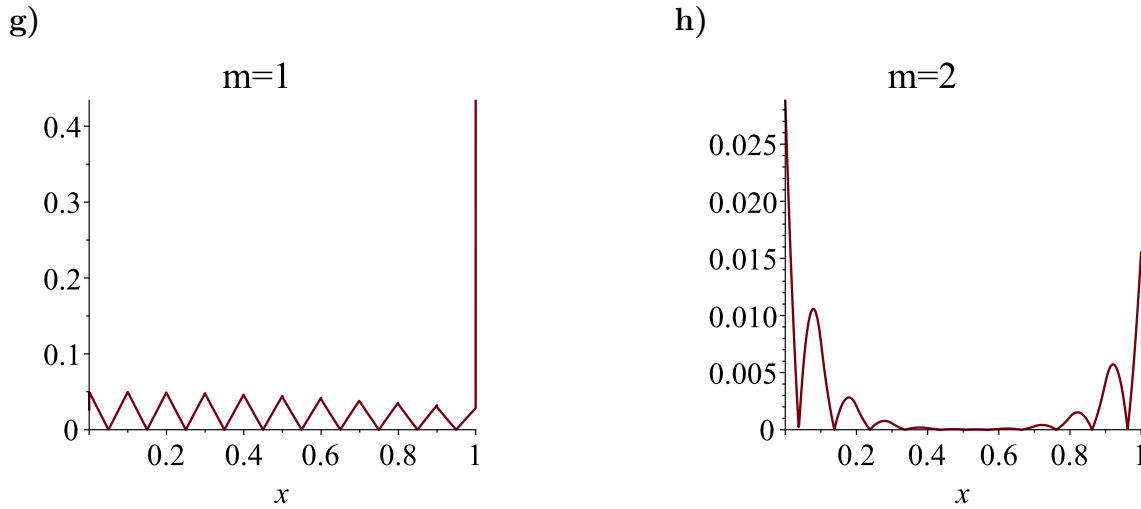


Figure 4 Graphs of absolute errors $|f_2(x) - S_m(f_2; x)|$ for $N = 10$ with $m = 1, 2$.

Table 1 Maximum absolute errors $\max |f_1(x) - S_m(x)|$ for $f_1(x) = e^x$.

N	$m = 1$	$m = 2$
$N = 10$	1.42488	0.07840
$N = 50$	1.37264	0.01569
$N = 100$	1.36591	0.00784

$= \sin x$, the error decreased from 0.02112 to 0.00288. In conclusion, transitioning from the $L_2^{(1)}(0, 1)$ space to the $L_2^{(2)}(0, 1)$ space allows for a reduction in error by several orders.

5 Conclusion

In this paper, we constructed the histosplines in the $L_2^{(1)}(0, 1)$ and $L_2^{(2)}(0, 1)$ spaces. Such type of histopolation method can be used in many different fields. The histospline is found by histograms can construct approximating function by using integral values on

Table 2 Maximum absolute errors $\max |f_2(x) - S_m(x)|$ for $f_2(x) = \sin x$.

N	$m = 1$	$m = 2$
$N = 10$	0.43492	0.02112
$N = 50$	0.42342	0.00577
$N = 100$	0.42209	0.00288

successive intervals instead of using function values at the knots. The obtained results show that the value of our approach. In the future studies, we aim to extend this approximation method by constructing histosplines in $L_2^{(m)}(0, 1)$ space. This method can be used to solve many practical problems. For example, it is possible to reconstruct a smooth precipitation graph if daily or monthly precipitation amounts are provided.

Declarations

Funding. The authors received no specific funding for this work.

Conflicts of interest. The authors declare no conflict of interest.

Author contributions. A.R. Hayotov: problem statement; M.M. Mukhtorova: numerical experiments.

Data availability. The data generated during this study are available from the authors on reasonable request.

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УДК 519.652.3

ФУНКЦИИ ГИСТОСПЛАЙНА ПРИ ВОССТАНОВЛЕНИИ ФУНКЦИЙ ПО ИНТЕГРАЛЬНЫМ ЗНАЧЕНИЯМ

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Актуальность: Восстановление непрерывных функций по их интегральным значениям на заданных интервалах является важной задачей прикладной и вычислительной математики, особенно в случаях, когда непосредственные значения функции в узлах неизвестны. Ряд исследователей проводили исследования в данной области, предлагая различные методы восстановления функций, включая традиционные подходы, алгебраические методы и другие.

Цель: Целью данной работы является исследование метода гистосплайнов в пространствах Соболева для восстановления непрерывных функций, заданных их интегральными значениями на интервалах, а также получение явных формул представления коэффициентов.

Метод: Метод гистосплайнов исследуется в пространствах Соболева для восстановления функций, заданных их интегральными значениями на интервалах. Показано, что при применении оператора дифференцирования к натуральному сплайну степень функции гистосплайна уменьшается на единицу.

Результаты: В результате в пространстве Соболева функций с квадратично интегрируемыми производными первого порядка на единичном интервале получается кусочно-постоянная функция, тогда как в пространстве Соболева функций с квадратично интегрируемыми производными второго порядка формируется квадратичная функция. Получены явные формулы для коэффициентов гистополирующего сплайна и представлены результаты численных экспериментов.

Практическая значимость: Построенные гистосплайны обеспечивают точное соответствие площадям соответствующих столбцов гистограммы, что позволяет создать эффективный подход к восстановлению функций в пространствах Соболева низких порядков.

Ключевые слова: аппроксимация, натуральный сплайн, гистограмма, гистополяция, гистосплайн

Цитирование: Хаётов А.Р., Мухторова М.М. Функции гистосплайна при восстановлении функций по интегральным значениям // Проблемы вычислительной и прикладной математики. – 2026. – № 4/1(75). – С. 5-15.

DOI: 10.71310/psam.4-1_75.2026.01

HISOBLASH VA AMALIY MATEMATIKA MUAMMOLARI

ПРОБЛЕМЫ ВЫЧИСЛИТЕЛЬНОЙ
И ПРИКЛАДНОЙ МАТЕМАТИКИ
PROBLEMS OF COMPUTATIONAL
AND APPLIED MATHEMATICS

ПРОБЛЕМЫ ВЫЧИСЛИТЕЛЬНОЙ И ПРИКЛАДНОЙ МАТЕМАТИКИ

№ 4/1(75) 2026

Журнал основан в 2015 году.

Издается 6 раз в год.

Учредитель:

Научно-исследовательский институт развития цифровых технологий и
искусственного интеллекта.

Главный редактор:

Равшанов Н.

Заместители главного редактора:

Арипов М.М., Шадиметов Х.М., Ахмедов Д.Д.

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Mascagni M. (США), Min A. (Германия), Singh M. (Южная Корея).

Журнал зарегистрирован в Агентстве информации и массовых коммуникаций при
Администрации Президента Республики Узбекистан.

Свидетельство №0856 от 5 августа 2015 года.

ISSN 2181-8460, eISSN 2181-046X

При перепечатке материалов ссылка на журнал обязательна.

За точность фактов и достоверность информации ответственность несут авторы.

Адрес редакции:

100125, г. Ташкент, м-в. Буз-2, 17А.

Тел.: +(998) 71 263-41-98.

Э-почта: journals@airi.uz.

Веб-сайт: <https://journals.airi.uz>.

Дизайн и вёрстка:

Шарипов Х.Д.

Отпечатано в типографии НИИ РЦТИИ.

Подписано в печать 05.10.2026 г.

Формат 60x84 1/8. Заказ № 5. Тираж 100 экз.

PROBLEMS OF COMPUTATIONAL AND APPLIED MATHEMATICS

No. 4/1(75) 2026

The journal was established in 2015.
6 issues are published per year.

Founder:

Digital Technologies and Artificial Intelligence Development Research Institute.

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Mascagni M. (USA), Min A. (Germany), Singh M. (South Korea).

The journal is registered by Agency of Information and Mass Communications under the
Administration of the President of the Republic of Uzbekistan.

Certificate of Registration No. 0856 of 5 August 2015.

ISSN 2181-8460, eISSN 2181-046X

At a reprint of materials the reference to the journal is obligatory.
Authors are responsible for the accuracy of the facts and reliability of the information.

Address:

100125, Tashkent, Buz-2, 17A.

Tel.: +(998) 71 263-41-98.

E-mail: journals@airi.uz.

Web-site: <https://journals.airi.uz>.

Layout design:

Sharipov Kh.D.

DTAIRI printing office.

Signed for print 05.10.2026

Format 60x84 1/8. Order No. 5. Print run of 100 copies.

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