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MATHEMATICAL MODELING OF SUSPENSION FILTRATION IN POROUS MEDIA

**Turakulov J.A., Juraev I.R., Tashtemirova N.N., Raxmatullayeva D.O.*

*jahondasturchi@gmail.com

Digital Technologies and Artificial Intelligence Development Research Institute,
17A, Buz-2, Tashkent, 100125 Uzbekistan

This article presents a mathematical model of suspension filtration in a filter section, taking into account the saturation of each layer of the porous medium. An incompressible fluid in free-flowing regions is described by the Darcy equations, and the pressure distribution is determined by the Poisson equation. The concentration in the suspension is solved together by the advection–diffusion equation taking into account the absorption. The model includes the change in porosity over time, allowing for the observation of the dynamics of changes in all parameters within this changed range. The conducted analysis showed that in the saturated part of the filter, conductivity and diffusion decrease, while the concentration remains unchanged in this range. These indicators help predict the full operating mode of the filter and select optimal parameters.

Keywords: mathematical modeling, suspension filtration, advection–diffusion equation, saturation, Darcy equation

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1 Introduction

The purification of liquids by filtration is a fundamental process in many industrial sectors, including pharmaceuticals, oil and gas production, and food processing. Suspension filtration in porous media is a complex hydrodynamic and physicochemical process, in which the internal absorption of suspension particles by porous media leads to a decrease in its absorption capacity. In the world, as freshwater resources decrease and the demand for industrial wastewater treatment increases, the development of mathematical models capable of predicting optimal filtration regimes is becoming increasingly important.

In [1], a mathematical model describing suspension filtration in active and passive porous zones with multi-stage particle deposition kinetics was proposed and solved numerically. The results showed that increasing the duration of irreversible deposition leads to the formation of a fully saturated region near the filter inlet, where deposition in the passive zone ceases while particle transport and deposition in the active zone continue. In [2], the authors numerically investigated coupled colmatation and suffusion processes in a single-layer porous medium, analyzing how pore-structure evolution influences filtration characteristics, velocity variation, and multilayer filtration regimes. The study established a theoretical and computational framework for modeling complex porous-medium filtration dynamics.

In [3] a comprehensive mathematical model and numerical algorithm were developed for filtration of liquids and ionized solutions by considering variable filtration velocity, concentration changes, cake-layer growth, gel-particle deposition, and hydraulic pressure increase. Numerical simulations demonstrated that the barodiffusion coefficient strongly

affects ion-exchange intensity and filter lifetime, while cake formation significantly alters flow rate, concentration, and pressure distribution. Study [4] examined filtration of liquid solutions containing fine solid particles and ionic components. A mathematical model and efficient computational algorithm were proposed to identify dominant process parameters and their admissible ranges, providing a scientific basis for optimizing industrial filtration conditions.

In [5], the microfiltration of silver nanoparticles through expanded polytetrafluoroethylene and polycarbonate membranes was analyzed numerically. The results revealed higher particle retention in isopropyl alcohol than in water, with capture efficiency increasing monotonically as particle size increased. Lower zeta-potential values were also observed in isopropyl alcohol compared to aqueous conditions. In [2, 6, 7], a three-phase mathematical model accounting for particle accumulation, porosity evolution, and permeability variation was introduced for suspension filtration in porous media. Numerical solutions obtained by the finite difference method showed good agreement with experimental data, confirming the reliability of the proposed model for predicting complex filtration processes.

The article [8, 9] performs mathematical and numerical modeling of laminar fluid flow in a sudden-expansion channel. For this purpose, the Navier–Stokes equations were solved using the finite difference method, and QUICK and implicit Upwind (US) computational schemes were used. In the study, the flow velocity, friction coefficient, and flow isoline values were determined for Reynolds numbers of 100, 400, 600, and 800, and the results were compared with experimental data. As a result, it was found that in small Reynolds numbers, the results of both numerical schemes were almost the same, and in large Reynolds numbers, the implicit scheme corresponds better to the experimental results than the QUICK scheme. Primary and secondary flow vortices formed in the channel were also identified.

In the article [10, 11], the authors considered the development of a three-dimensional model and a numerical algorithm based on the method of coordinate separation. This problem is intended for the analysis and prediction of gas filtration in porous media. The method of coordinate separation made it possible to reduce the calculation time by reducing the initial problem to three local one-dimensional problems and reducing the number of calculation steps. The mathematical model and numerical algorithm allow for the selection of optimal modes for the design or development of gas fields.

The foregoing review of the literature demonstrates that accurate numerical prediction of particle transport in flowing liquids remains an open and practically significant challenge both in fundamental research and industrial applications.

2 Problem statement

The mathematical model proposed below combines Darcy's law with the advection–diffusion–absorption equations. For a suspension percolating through a porous medium, this coupled system makes it possible to track the simultaneous evolution of particle concentration and medium structure in both space and time [1, 3, 5].

$$\begin{aligned} \frac{\partial(m\theta)}{\partial t} + W_x \frac{\partial\theta}{\partial x} + W_y \frac{\partial\theta}{\partial y} &= \frac{\partial}{\partial x} \left(D_{\text{eff}} m \frac{\partial\theta}{\partial x} \right) + \frac{\partial}{\partial y} \left(D_{\text{eff}} m \frac{\partial\theta}{\partial y} \right) - \frac{\partial\sigma}{\partial t}, \\ \frac{\partial\sigma}{\partial t} &= \lambda_0 \cdot \left(1 - \frac{\sigma}{\sigma_{\text{max}}} \right) \cdot \sqrt{W_x^2 + W_y^2} \cdot \theta, \end{aligned} \quad (1)$$

where the velocity and pressure fields satisfy

$$\begin{aligned} \frac{\partial W_x}{\partial x} + \frac{\partial W_y}{\partial y} &= 0, \quad W_x = -\frac{K(m)}{\mu} \frac{\partial P}{\partial x}, \quad W_y = -\frac{K(m)}{\mu} \frac{\partial P}{\partial y}, \\ \frac{\partial}{\partial x} \left(\frac{K(m)}{\mu} \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{K(m)}{\mu} \frac{\partial P}{\partial y} \right) &= 0, \\ K(m) &= K_0 \frac{m^3}{(1-m)^2} \frac{(1-m_0)^2}{m_0^3}, \quad D_{\text{eff}} = D \left(\frac{m}{m_0} \right)^{1.5}, \quad m = m_0 - \frac{\sigma}{\rho_z}. \end{aligned} \quad (2)$$

Initial and boundary conditions:

$$\begin{aligned} \theta|_{t=0} &= \theta_0, \quad \sigma|_{t=0} = 0, \\ \theta|_{x=0} &= \theta^0, \quad \frac{\partial \theta}{\partial x} \Big|_{x=L_x} = 0, \quad \frac{\partial \theta}{\partial y} \Big|_{y=0} = 0, \quad \frac{\partial \theta}{\partial y} \Big|_{y=L_y} = 0, \\ \frac{K(m)}{\mu} \frac{\partial P}{\partial x} \Big|_{x=0} &= -W_x, \quad \frac{\partial P}{\partial x} \Big|_{x=L_x} = 0, \quad \frac{\partial P}{\partial y} \Big|_{y=0} = 0, \quad \frac{\partial P}{\partial y} \Big|_{y=L_y} = 0, \\ W_x|_{x=0} &= W_x^0, \quad \frac{\partial W_x}{\partial x} \Big|_{x=L_x} = 0, \quad W_x|_{y=0} = 0, \quad W_x|_{y=L_y} = 0, \\ W_y|_{x=0} &= 0, \quad \frac{\partial W_y}{\partial x} \Big|_{x=L_x} = 0, \quad W_y|_{y=0} = 0, \quad W_y|_{y=L_y} = 0. \end{aligned} \quad (3)$$

$$(4)$$

Here θ is the concentration, W is the flow velocity, D is the diffusion coefficient, ρ_z is the particle density in the suspension, μ is the dynamic viscosity, P is the pressure, σ is the mass accumulated in the porous medium, m is the porosity, K is the permeability, and λ_0 is the filtration rate.

Since it is impossible to analytically solve the above mathematical model, results are obtained by numerical solution. The following grid area is entered for numerical solution.

$$\Omega = \{x_i = i\Delta x, y_j = j\Delta y; t^n = n\Delta t; \quad i \in 1, 2, \dots, I; \quad j \in 1, 2, \dots, J; \quad i, j, n \in N\}$$

The mathematical model is then discretized using an explicit finite difference scheme.

$$\begin{aligned} & \frac{(m\theta)_{i,j}^{n+1} - (m\theta)_{i,j}^n}{\Delta t} + W_{xi,j}^n \frac{\theta_{i,j}^n - \theta_{i-1,j}^n}{\Delta x} + W_{yi,j}^n \frac{\theta_{i,j}^n - \theta_{i,j-1}^n}{\Delta y} = \\ & = \left[\frac{(D_{\text{eff}}m)_{i+1/2,j}^n (\theta_{i+1,j}^n - \theta_{i,j}^n) - (D_{\text{eff}}m)_{i-1/2,j}^n (\theta_{i,j}^n - \theta_{i-1,j}^n)}{\Delta x^2} + \right. \\ & \left. + \frac{(D_{\text{eff}}m)_{i,j+1/2}^n (\theta_{i,j+1}^n - \theta_{i,j}^n) - (D_{\text{eff}}m)_{i,j-1/2}^n (\theta_{i,j}^n - \theta_{i,j-1}^n)}{\Delta y^2} \right] - \frac{\sigma_{i,j}^{n+1} - \sigma_{i,j}^n}{\Delta t}, \\ & \frac{\sigma_{i,j}^{n+1} - \sigma_{i,j}^n}{\Delta t} = \lambda_0 \cdot \left(1 - \frac{\sigma_{i,j}^n}{\sigma_{\text{max}}} \right) \cdot \sqrt{(W_{xi,j}^n)^2 + (W_{yi,j}^n)^2} \cdot \theta_{i,j}^n, \\ & W_{xi,j}^n = -\frac{K(m_{i,j}^n)}{\mu} \frac{(P_{i,j}^n - P_{i-1,j}^n)}{\Delta x}, \quad W_{yi,j}^n = -\frac{K(m_{i,j}^n)}{\mu} \frac{(P_{i,j}^n - P_{i,j-1}^n)}{\Delta y}, \end{aligned}$$

$$\begin{aligned}
& \frac{K(m_{i+1/2,j}^n) (P_{i+1,j}^n - P_{i,j}^n) - K(m_{i-1/2,j}^n) (P_{i,j}^n - P_{i-1,j}^n)}{\mu \Delta x^2} + \\
& + \frac{K(m_{i,j+1/2}^n) (P_{i,j+1}^n - P_{i,j}^n) - K(m_{i,j-1/2}^n) (P_{i,j}^n - P_{i,j-1}^n)}{\mu \Delta y^2} = 0, \\
K(m) &= K_0 \frac{(m_{i,j}^n)^3}{(1 - m_{i,j}^n)^2} \frac{(1 - m_0)^2}{m_0^3}, \quad D_{\text{eff}} = D \left(\frac{m_{i,j}^n}{m_0} \right)^{1.5}, \quad m_{i,j}^n = m_0 - \frac{\sigma_{i,j}^n}{\rho_z}. \quad (5)
\end{aligned}$$

Simplify the above numerically discretized equations

$$\begin{aligned}
& \theta_{i,j}^{n+1} = \\
& = \frac{1}{m_{i,j}^{n+1}} \left[(m\theta)_{i,j}^n + \frac{\Delta t}{m_{i,j}^n} \left(\frac{(D_{\text{eff}}m)_{i+1/2,j}^n (\theta_{i+1,j}^n - \theta_{i,j}^n)}{\Delta x^2} - \frac{(D_{\text{eff}}m)_{i-1/2,j}^n (\theta_{i,j}^n - \theta_{i-1,j}^n)}{\Delta x^2} + \right. \right. \\
& \quad + \frac{(D_{\text{eff}}m)_{i,j+1/2}^n (\theta_{i,j+1}^n - \theta_{i,j}^n)}{\Delta y^2} - \frac{(D_{\text{eff}}m)_{i,j-1/2}^n (\theta_{i,j}^n - \theta_{i,j-1}^n)}{\Delta y^2} - \\
& \quad \left. \left. - W_{xi,j}^n \frac{\theta_{i,j}^n - \theta_{i-1,j}^n}{\Delta x} - W_{yi,j}^n \frac{\theta_{i,j}^n - \theta_{i,j-1}^n}{\Delta y} - \frac{\sigma_{i,j}^{n+1} - \sigma_{i,j}^n}{\Delta t} \right) \right], \quad (6)
\end{aligned}$$

$$\sigma_{i,j}^{n+1} = \sigma_{i,j}^n + \Delta t \lambda_0 \cdot \left(1 - \frac{\sigma_{i,j}^n}{\sigma_{\text{max}}} \right) \cdot \sqrt{(W_{xi,j}^n)^2 + (W_{yi,j}^n)^2} \cdot \theta_{i,j}^n,$$

$$\begin{aligned}
& P_{i,j}^n = \\
& = \frac{K(m_{i+1/2,j}^n) P_{i+1,j}^n + K(m_{i-1/2,j}^n) P_{i-1,j}^n + K(m_{i,j+1/2}^n) P_{i,j+1}^n + K(m_{i,j-1/2}^n) P_{i,j-1}^n}{K(m_{i+1/2,j}^n) + K(m_{i-1/2,j}^n) + K(m_{i,j+1/2}^n) + K(m_{i,j-1/2}^n)}, \\
W_{xi,j}^n &= -\frac{K(m_{i,j}^n) (P_{i,j}^n - P_{i-1,j}^n)}{\mu \Delta x}, \quad W_{yi,j}^n = -\frac{K(m_{i,j}^n) (P_{i,j}^n - P_{i,j-1}^n)}{\mu \Delta y}, \\
K(m_{i,j}^n) &= K_0 \frac{(m_{i,j}^n)^3}{(1 - m_{i,j}^n)^2} \frac{(1 - m_0)^2}{m_0^3}, \quad D_{\text{eff}} = D \left(\frac{m_{i,j}^n}{m_0} \right)^{1.5}, \quad m_{i,j}^n = m_0 - \frac{\sigma_{i,j}^n}{\rho_z}.
\end{aligned}$$

Numerical stability of the explicit scheme is ensured by enforcing the Courant–Friedrichs–Lewy (CFL) condition

$$\Delta t < \min \left(\frac{\Delta x}{W_x}, \frac{\Delta y}{W_y}, \frac{\Delta x^2 + \Delta y^2}{2D_{\text{eff}}} \right). \quad (7)$$

3 Results and discussion

To examine the spatial distribution of concentration and the hydrodynamic characteristics of the flow, numerical simulations were performed on a computational cluster; the results are presented in Figs. 1–6.

Computational experiments were performed using the following initial parameter values [6]:

$$\begin{aligned}
P_0 &= 2 \cdot 10^5 \text{ Pa}, \quad \theta_0 = 1.0 \text{ kg/m}^3, \quad W_x = 0.01 \text{ m/s}, \quad W_y = 0, \\
\lambda_0 &= 2.0, \quad \sigma_{\max} = 0.05, \quad \rho_z = 2500 \text{ kg/m}^3, \quad \mu = 10^{-3} \text{ Pa} \cdot \text{s}, \\
D &= 10^{-6} \text{ m}^2/\text{s}, \quad m_0 = 0.4, \quad K_0 = 10^{-10}.
\end{aligned}$$

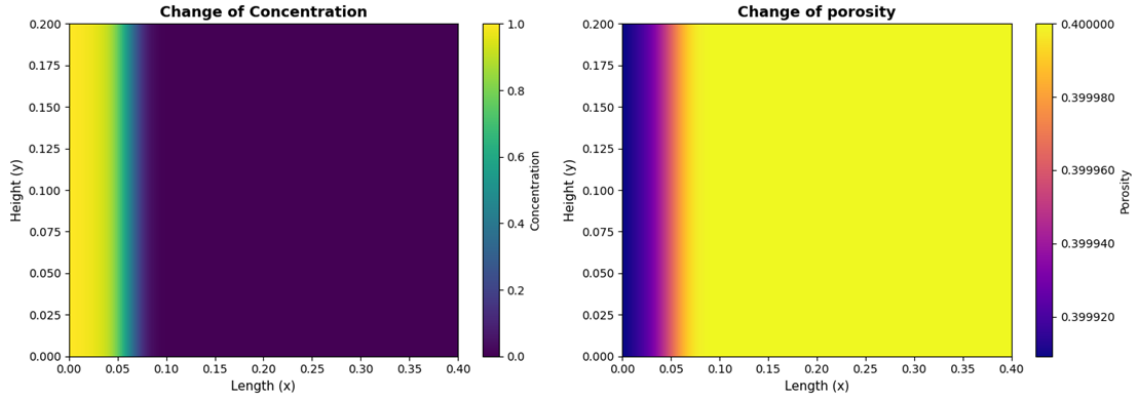


Figure 1 Change in concentration and medium porosity over 1 hour.

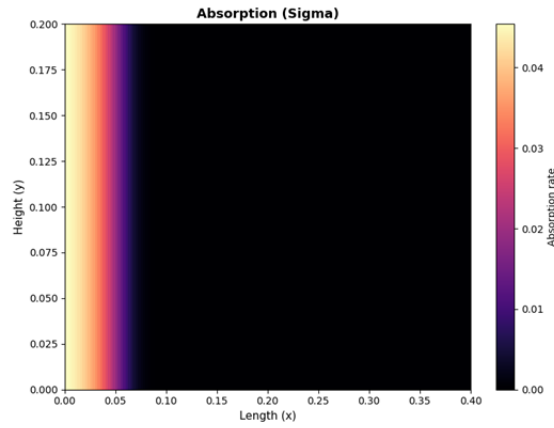


Figure 2 Change in absorption rate over 1 hour.

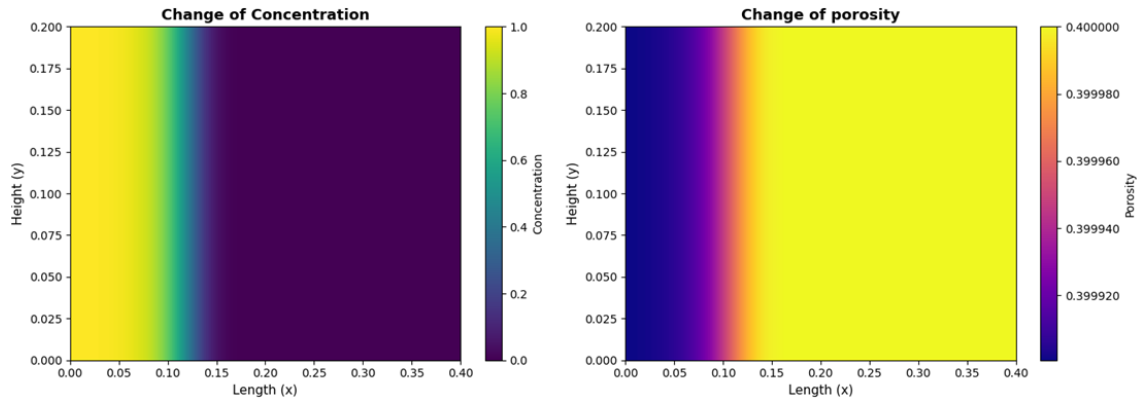


Figure 3 Change in concentration and medium porosity over 9 hours.

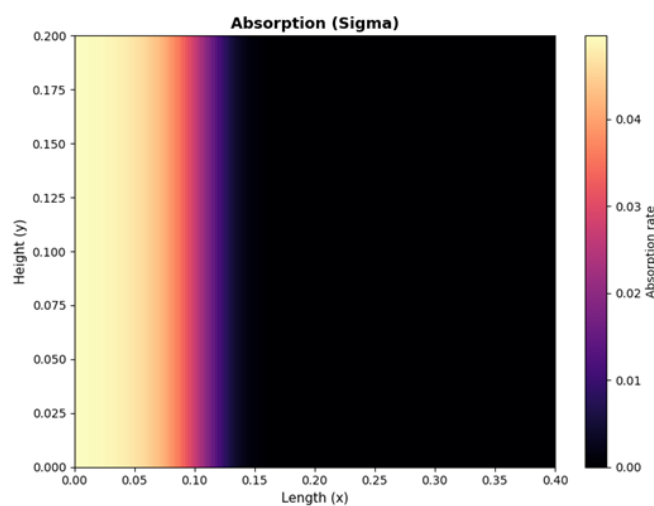


Figure 4 Change in absorption rate over 9 hours.

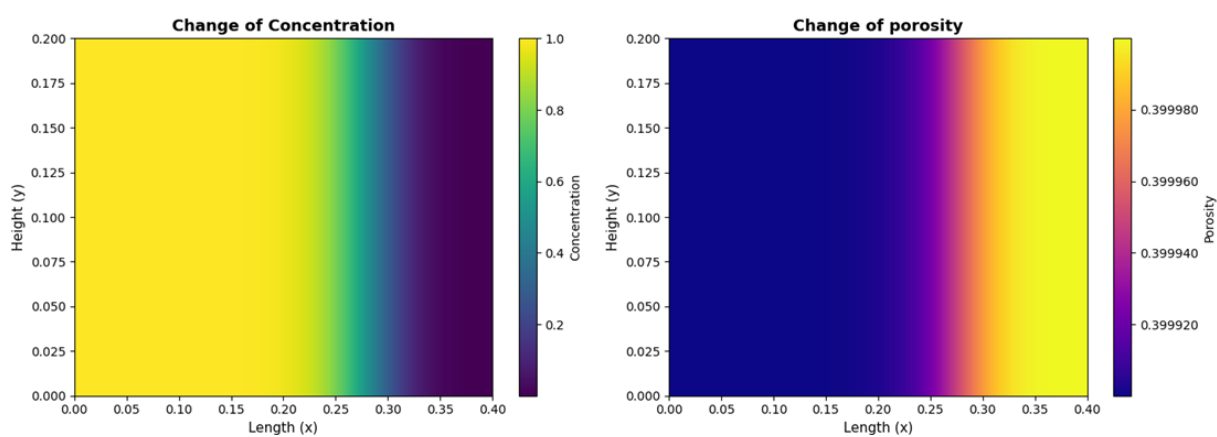


Figure 5 Change in concentration and medium porosity over 12 hours.

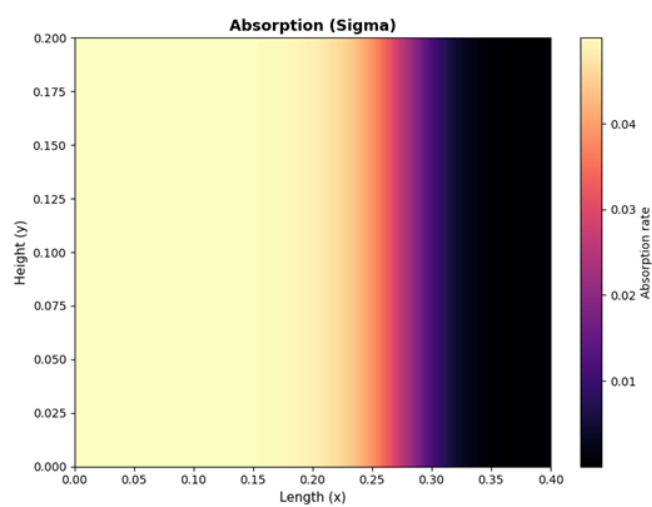


Figure 6 Change in absorption rate over 12 hours.

The results show that the concentration front gradually shifts from the inlet to the outlet over time. Initially, the accumulation of particles occurs near the inlet zone, and subsequently, suspended particles penetrate deeper into the porous medium. At the same time, changes in porosity can be observed in areas with high particle deposition. In this area, the concentration is absorbed to maximum saturation. Mainly because this condition causes internal saturation, a small change in porosity can be observed. These results confirm the strong interaction between particle transport and structural changes in the porous medium during filtration.

4 Conclusion

The results obtained show that the filtration process depends on particle transport and the saturation capacity of the porous medium. When suspended particles move in a porous medium, their absorption reduces the absorption capacity of the porous medium in that zone. Over time, the concentration passes through the saturated part of the medium without changing. This indicates the nonlinear nature of suspension filtration and its influence on internal saturation. The developed mathematical model predicts the dynamic changes occurring in the porous medium during the filtration process with high precision.

Declarations

Conflicts of interest. The authors declare no conflict of interest.

Data availability. The data are available from the authors on reasonable request.

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МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ФИЛЬТРАЦИИ СУСПЕНЗИИ В ПОРИСТЫХ СРЕДАХ

**Туракулов Ж.А., Жураев И.Р., Таштемирова Н.Н., Рахматуллаева Д.О.*

**jahondasturchi@gmail.com*

Научно-исследовательский институт развития цифровых технологий и искусственного интеллекта,
100125, Узбекистан, г. Ташкент, м-в Буз-2, 17А

В данной статье представлена математическая модель фильтрации суспензии в фильтровальном сечении с учетом насыщения каждого слоя пористой среды. Несжимаемая жидкость в свободнотекущих областях описывается уравнениями Дарси, а распределение давления определяется уравнением Пуассона. Концентрация в суспензии решается совместно адвективно-диффузионным уравнением с учетом поглощения. Модель включает изменение пористости во времени, что позволяет наблюдать динамику изменений всех параметров в этом измененном диапазоне. Проведенный анализ показал, что в насыщенной части фильтра проводимость и диффузия снижаются, в то время как концентрация в этом диапазоне остается неизменной. Эти показатели помогают прогнозировать полный режим работы фильтра и выбрать оптимальные параметры.

Ключевые слова: математическое моделирование, фильтрация суспензии, адвективно-диффузионное уравнение, насыщение, уравнение Дарси

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