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DYNAMICAL ANALYSIS OF A STOCHASTIC SICR TRANSMISSION MODEL FOR POTATO EARLY AND LATE BLIGHT

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As a devastating fungal disease that seriously threatens potato production, the transmission process of potato early and late blight is significantly affected by environmental fluctuations. To accurately reveal the epidemic regularity of this disease, a stochastic SICR transmission model for potato early and late blight was constructed in this paper. Through dynamical analysis, first, it was proved that the system has a unique global positive solution, which ensures the biological rationality of the model; second, based on Has'minskii's theory, the sufficient conditions for the existence of the ergodic stationary distribution of the model were derived, clarifying the dynamical mechanism underlying the persistent epidemic of the disease; third, the threshold conditions for disease extinction and persistence in mean were established, identifying the key nodes for disease prevention and control. The results show that when $R_0^s > 1$, the model has an ergodic stationary distribution and the disease persists; when $\bar{R}_0 < 1$, the disease will eventually become extinct. This study provides important theoretical support for the risk early warning and precise prevention and control of potato early and late blight.

Keywords: potato early and late blight, stochastic SICR model, stationary distribution, disease extinction, disease persistence

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1 Introduction

Potato, the world's fourth major staple crop, underpins the steady development of food security and agricultural economy [1]. Among various potato diseases, early blight and late blight are the most prevalent and destructive, which are caused by *Alternaria solani* and *Phytophthora infestans*, respectively [2, 3]. These diseases are highly contagious and prone to large-scale outbreaks, directly reducing potato yields by 20% to 50% in most cases, and even resulting in total yield loss in severely affected fields [4–6]. The epidemic spread of the diseases is governed by multiple factors. It is influenced by intrinsic biological factors such as pathogen infectivity and host resistance, as well as external fluctuations of environmental variables including temperature, humidity and precipitation [7–9]. For this reason, traditional deterministic models cannot accurately predict their transmission patterns.

Mathematical modeling serves as a core method for researching the transmission dynamics of infectious diseases and plant diseases. In recent years, dynamic models have become a mainstream tool for analyzing disease transmission mechanisms [10]. Song et al. established a viral infection dynamic model incorporating the saturated infection rate parameter [11]. Li et al. investigated the dynamic behaviors of the hepatitis B virus infection model where hepatocytes follow the logistic growth pattern [12]. Given the distinct staged characteristics of disease transmission, Qiao et al. first proposed the SICR epidemic model, which divides the population into susceptible, acutely infected, pathogen-carrier and recovered groups. This work provides a new paradigm for the modeling and analysis of staged diseases [13]. Although this model has been successfully applied to various diseases with latent infection and pathogen carriage [14, 15], it has not yet been adopted in the research on potato early blight and late blight.

The transmission of potato early blight and late blight follows distinct staged patterns. Once healthy susceptible plants are exposed to pathogens, they will develop typical acute symptoms and enter the acutely infected stage. Some infected plants then turn into latently infected individuals and become pathogen carriers. In addition, many diseased plants can gradually recover with the help of their own immunity or manual prevention and control measures. The above evolutionary process of the diseases is highly consistent with the framework of the SICR model. Under field conditions, meteorological factors such as temperature, humidity and rainfall fluctuate randomly, which directly affects the infectivity and spreading rate of pathogens [16–21]. The traditional deterministic SICR model fails to take such random disturbances into account, thus leading to deviations in simulation and prediction results. Therefore, introducing random perturbation terms to establish a stochastic transmission model for potato early blight and late blight can effectively improve the prediction accuracy of disease epidemics and has important research significance.

Drawing on the ideas and achievements of existing epidemic transmission models, this paper addresses the limitations of traditional deterministic models. By incorporating random environmental fluctuations into the research on the spread of potato early blight and late blight, a stochastic SICR transmission model is established. Dynamic analysis is adopted to clarify the dynamic laws such as disease extinction and persistent prevalence under variable environmental conditions, which provides an important theoretical reference for the formulation and optimization of subsequent epidemic prevention and control strategies.

The overall structure of this paper is organized as follows. Section 2 establishes the stochastic SICR model for potato early blight and late blight, and verifies the existence and uniqueness of its global positive solutions. Section 3 discusses the existence of the ergodic stationary distribution of the model. Section 4 further derives the criteria for disease extinction and persistence in mean. Section 5 summarizes the research findings and presents prospects for future studies.

2 Model construction and existence of a global positive solution

2.1 Model assumptions and construction

Combined with the transmission characteristics of potato early and late blight, the following assumptions are made:

- the potato population is divided into four categories: susceptible plants ($S(t)$), acutely infected plants ($I(t)$), pathogen-carrying plants ($C(t)$), and recovered plants ($R(t)$), among which recovered plants have immunity;

- μ represents the natural growth rate and mortality rate of the total population;
- γ_1 represents the recovery (or detachment) transition rate of acutely infected plants;
- γ_2 represents the transition rate from pathogen-carrying plants to recovered plants;
- β represents the contact rate between susceptible plants and acutely infected plants or pathogen-carrying plants;
- $\alpha \in [0,1]$ represents the increased transmission rate of acutely infected plants compared with pathogen-carrying plants;
- $q \in [0,1]$ represents the proportion of acutely infected plants transitioning to pathogen-carrying plants.

Based on the above assumptions, the SICR model is established as follows:

$$\begin{cases} \dfrac{dS(t)}{dt} = \mu - \beta S(t)I(t) - \alpha\beta S(t)C(t) - \mu S(t), \\ \dfrac{dI(t)}{dt} = \beta S(t)I(t) + \alpha\beta S(t)C(t) - \gamma_1 I(t) - \mu I(t), \\ \dfrac{dC(t)}{dt} = q\gamma_1 I(t) - \gamma_2 C(t) - \mu C(t), \\ \dfrac{dR(t)}{dt} = (1-q)\gamma_1 I(t) + \gamma_2 C(t) - \mu R(t), \end{cases} \tag{1}$$

Since the fourth equation of system (1) is decoupled (it does not affect the dynamics of the first three equations), we can simplify system (1) to the following subsystem:

$$\begin{cases} \dfrac{dS(t)}{dt} = \mu - \beta S(t)I(t) - \alpha\beta S(t)C(t) - \mu S(t), \\ \dfrac{dI(t)}{dt} = \beta S(t)I(t) + \alpha\beta S(t)C(t) - \gamma_1 I(t) - \mu I(t), \\ \dfrac{dC(t)}{dt} = q\gamma_1 I(t) - \gamma_2 C(t) - \mu C(t), \end{cases} \tag{2}$$

The initial condition of system (2) is $(S(0), I(0), C(0)) \in {\mathbb{R}_+}^3$.

According to the theory in [13], system (2) has the following properties. The set $\Gamma = \{(S(t), I(t), C(t)) \in {\mathbb{R}_+}^3 : S(t) + I(t) + C(t) \leqslant 1\}$ is positively invariant for (2). The disease-free equilibrium $E_0 = (S_0, I_0, C_0) = (1, 0, 0)$, which always exists, is globally asymptotically stable if $R_0 = \dfrac{\beta}{\gamma_1 + \mu} + \dfrac{q\gamma_1}{\gamma_1 + \mu}\dfrac{\alpha\beta}{\gamma_2 + \mu} \leqslant 1$.

When $R_0 > 1$, system (2) has a unique positive equilibrium E^* and it is globally asymptotically stable. However, in practical ecological environments, all biological mathematical models are inevitably disturbed by random environmental noise [22–26], which cannot be ignored in real-world modeling. Therefore, establishing epidemic dynamic models based on stochastic differential equations can more accurately and objectively reflect the evolutionary rules of infectious diseases and improve the reliability of dynamic prediction. Against this research background, many scholars at home and abroad introduce random perturbations into classical deterministic models to describe the effects of natural environmental fluctuations on disease transmission. For instance, Zhang, Jiang, Hayat and Ahmad systematically investigated the dynamic behaviors of the stochastic SVIR model [19]. Wang, Jiang, Alsaedi and Hayat studied a stochastic HIV model with Logistic target cell growth and a nonlinear immune response function. They not only derived the criteria for virus extinction, but also proved the existence of the ergodic stationary distribution [27]. For more relevant studies, readers may refer to [28–35].

Inspired by the aforementioned research work, this paper introduces stochastic perturbations into Model (2). Considering the uncertainty of environmental factors such as temperature, humidity, and precipitation in the field, white noise is used to simulate such environmental perturbations, and these stochastic perturbations are proportional to the variables $S(t), I(t), C(t)$ in System (2). Thus, the stochastic version corresponding to System (2) has the following form:

$$\begin{cases} dS(t) = [\mu - \beta S(t)I(t) - \alpha\beta S(t)C(t) - \mu S(t)]dt + \sigma_1 S(t)dB_1(t), \\ dI(t) = [\beta S(t)I(t) + \alpha\beta S(t)C(t) - \gamma_1 I(t) - \mu I(t)]dt + \sigma_2 I(t)dB_2(t), \\ dC(t) = [q\gamma_1 I(t) - \gamma_2 C(t) - \mu C(t)]dt + \sigma_3 C(t)dB_3(t), \end{cases} \tag{3}$$

where $B_i(t)$, $i = 1, 2, 3$, are the white noises, representing the impact of temperature, humidity, and precipitation fluctuations on the fungal transmission rate. For example, late blight spreads extremely rapidly when humidity exceeds 90%, and such environmental fluctuations are reflected as random terms in the model. In other words, $B = (B_1(t), B_2(t), B_3(t))$, $t \geqslant 0$, is a vector of mutually independent standard Brownian motions defined on a complete probability space $(\Omega, \mathcal{F}, P)$ with a filtration $\{\mathcal{F}_t\}_{t\geqslant 0}$ satisfying the general conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all P-null sets), $\sigma_1, \sigma_2, \sigma_3$ stand for the intensities of the white noise, $\sigma_i > 0$ $(i = 1, 2, 3)$. All the other parameters have the same meaning as that of the system (2).

2.2 Existence and uniqueness of the global positive solution

This section aims to prove that the solution of system (3) is globally existent and positive for any given initial value. In general, to verify that a stochastic differential equation has a unique global solution without explosion in finite time for arbitrary initial values, the coefficients of the system are required to satisfy both the local Lipschitz condition and the linear growth condition [36]. Although the coefficients of system (3) satisfy the local Lipschitz continuity, they fail to meet the linear growth condition, which implies that the solution may blow up in finite time theoretically. In view of this, we will rigorously prove that system (3) admits a unique global positive solution for all initial values in the following content.

Theorem 1. *For any initial value $(S(0), I(0), C(0)) \in {\mathbb{R}_+}^3$, there is a unique positive solution $(S(t), I(t), C(t))$ on $t \geqslant 0$ for system (3); further, the solution will remain in ${\mathbb{R}_+}^3$ with probability one, in brief, $(S(t), I(t), C(t)) \in {\mathbb{R}_+}^3$ for all $t \geqslant 0$ almost surely (a.s.).*

Proof. Since the coefficients of system (3) satisfy the local Lipschitz condition, system (3) has, for any initial value, a unique local solution $(S(t), I(t), C(t))$ on $t \in [0, \tau_e)$, where τ_e is the explosion time [37, 38]. Next, to show that this solution is global, we need to verify that $\tau_e = \infty$ a.s. Let $k_0 \geqslant 1$ be large enough such that $S(0), I(0), C(0)$ all lie within the interval $[\frac{1}{k_0}, k_0]$. For each integer $k \geqslant k_0$, define the stopping time

$$\tau_k = \inf\left\{t \in [0, \tau_e) : S(t) \notin \left(\frac{1}{k}, k\right) \text{ or } I(t) \notin \left(\frac{1}{k}, k\right) \text{ or } C(t) \notin \left(\frac{1}{k}, k\right)\right\},$$

where throughout this paper, we set $\inf \varnothing = \infty$ (as usual $\varnothing$ is the empty set). Clearly τ_k is increasing when $k \to \infty$. Let $\tau_\infty = \lim\limits_{k\to\infty} \tau_k$, whence $\tau_\infty \leqslant \tau_e$ a.s. If we can show $\tau_\infty = \infty$, then $\tau_e = \infty$, and $(S(t), I(t), C(t)) \in {\mathbb{R}_+}^3$ a.s. for all $t \geqslant 0$. That shows that we are requested to illustrate that $\tau_\infty = \infty$, a.s. in order to finish the proof. If this assertion is not true, then there is a pair of constants $T > 0$ and $\varepsilon \in (0,1)$ such that $P\{\tau_\infty \leqslant T\} > \varepsilon$. Hence there is an integer $k_1 \geqslant k_0$ such that for all $k \geqslant k_1$

$$P\{\tau_k \leqslant T\} \geqslant \varepsilon. \tag{4}$$

Define a C^2-function $V : {\mathbb{R}_+}^3 \to \mathbb{R}_+$ by

$$V(S,I,C) = (S - b - b\ln\frac{S}{b}) + (I - 1 - \ln I) + fC,$$

where b, f are positive constants to be determined later. The non negativity of this function can be attained from $u - 1 - \ln u \geqslant 0$ for any $u > 0$. Applying the Itô formula to V, we get

$$\begin{aligned} dV &= (1-\frac{b}{S})dS + (1-\frac{1}{I})dI + fdC + \frac{1}{2}\frac{b}{S^2}(dS)^2 + \frac{1}{2}\frac{1}{I^2}(dI)^2 = \\ &= (1-\frac{b}{S})[(\mu - \beta SI - \alpha\beta SC - \mu S)dt + \sigma_1 S dB_1(t)] + \frac{1}{2}b\sigma_1^2 dt + \\ &+(1-\frac{1}{I})[(\beta SI + \alpha\beta SC - \gamma_1 I - \mu I)dt + \sigma_2 I dB_2(t)] + \frac{1}{2}\sigma_2^2 dt + \\ &+f[(q\gamma_1 I - \gamma_2 C - \mu C)dt + \sigma_3 C dB_3(t)] = \\ &= LVdt + \sigma_1(S-b)dB_1(t) + \sigma_2(I-1)dB_2(t) + f\sigma_3 C dB_3(t). \end{aligned} \tag{5}$$

where LV is defined by

$$\begin{aligned} LV &= (1-\frac{b}{S})(\mu - \beta SI - \alpha\beta SC - \mu S) + \frac{1}{2}b\sigma_1^2 + (1-\frac{1}{I})(\beta SI + \\ &+\alpha\beta SC - \gamma_1 I - \mu I) + \frac{1}{2}\sigma_2^2 + f(q\gamma_1 I - \gamma_2 C - \mu C) = \\ &= (b+2)\mu - (\mu+\beta)S - \frac{b\mu}{S} - \frac{\alpha\beta SC}{I} + \gamma_1 + \frac{1}{2}b\sigma_1^2 + \frac{1}{2}\sigma_2^2 + \\ &+(b\beta + fq\gamma_1 - \gamma_1 - \mu)I + (b\alpha\beta - f\gamma_2 - f\mu)C \leqslant \\ &\leqslant (b+2)\mu + \gamma_1 + \frac{1}{2}b\sigma_1^2 + \frac{1}{2}\sigma_2^2 + (b\beta + fq\gamma_1 - \gamma_1 - \mu)I + (b\alpha\beta - f\gamma_2 - f\mu)C. \end{aligned}$$

We choose $b = \dfrac{(\gamma_1+\mu)(\gamma_2+\mu)}{\beta(\gamma_2+\mu+\alpha q\gamma_1)}$ and $f = \dfrac{\alpha(\gamma_1+\mu)}{\gamma_2+\mu+\alpha q\gamma_1}$ such that $(b\beta + fq\gamma_1 - \gamma_1 - \mu)I = 0$ and $(b\alpha\beta - f\gamma_2 - f\mu)C = 0$, then we can attain

$$LV \leqslant (b+2)\mu + \gamma_1 + \frac{1}{2}b\sigma_1^2 + \frac{1}{2}\sigma_2^2 := K. \tag{6}$$

where K is a positive constant. So we have

$$dV \leqslant Kdt + \sigma_1(S-b)dB_1(t) + \sigma_2(I-1)dB_2(t) + f\sigma_3 C dB_3(t). \tag{7}$$

Integrating (7) from 0 to $\tau_k \wedge T$ and then taking the expectation on both sides, further according to Theorem 1.5.8 (ii) in [36, 37], we can obtain:

$$EV(S(\tau_k \wedge T), I(\tau_k \wedge T), C(\tau_k \wedge T)) \leqslant V(S(0), I(0), C(0)) + KE(\tau_k \wedge T).$$

Therefore,

$$EV(S(\tau_k \wedge T), I(\tau_k \wedge T), C(\tau_k \wedge T)) \leqslant V(S(0), I(0), C(0)) + KT. \tag{8}$$

Set $\Omega_k = \{\tau_k \leqslant T\}$ for $k \geqslant k_1$ and by means of (4), we can obtain $P(\Omega_k) \geqslant \varepsilon$. Notice that for every $\omega \in \Omega_k$, there exists that $S(\tau_k, \omega)$ or $I(\tau_k, \omega)$ or $C(\tau_k, \omega)$ equals either k or $\frac{1}{k}$. Therefore, $V(S(\tau_k, \omega), I(\tau_k, \omega), C(\tau_k, \omega))$ is no less than either $k - 1 - \ln k$ or $\frac{1}{k} - 1 + \ln k$.

Consequently, we have

$$V(S(\tau_k, \omega), I(\tau_k, \omega), C(\tau_k, \omega)) \geqslant (k - 1 - \ln k) \wedge \left(\frac{1}{k} - 1 + \ln k\right) := H(k).$$

According to (8), we can attain:

$$\begin{aligned} V(S(0), I(0), C(0)) + KT &\geqslant E(I_{\Omega_k}(\omega) V(S(\tau_k \wedge T), I(\tau_k \wedge T), C(\tau_k \wedge T))) \geqslant \\ &\geqslant P(\Omega_k(\omega)) H(k) \geqslant \\ &\geqslant \varepsilon H(k), \end{aligned}$$

where I_{Ω_k} stands for the indicator function of Ω_k. Letting $k \to \infty$, then

$$\infty > V(S(0), I(0), C(0)) + KT \geqslant \varepsilon H(k) \to \infty,$$

which leads to a contradiction. Thus the proof is completed.

3 Existence of an ergodic stationary distribution of system (3)

In the study of epidemic dynamics of potato plants, it is the core objective to clarify the outbreak timing and persistent transmission rules of early blight caused by *Alternaria solani* and late blight induced by *Phytophthora infestans* in farmland. Within the framework of deterministic dynamic models, analyzing the dynamic properties of the endemic equilibrium is the key to exploring the evolutionary laws of disease prevalence. Specifically, rigorous theoretical derivation can be adopted to verify the global asymptotic stability and global attractivity of the positive equilibrium of the system, so as to reveal the prevalence and persistence mechanism of potato early blight and late blight.

However, the stochastic system (3) has no endemic equilibrium in the classical sense. Accordingly, based on the theory proposed by Has'minskii [38], this section derives the conditions for the existence of the ergodic stationary distribution of the model. We first introduce the fundamental theory related to stationary distributions, which serves as an important basis for the subsequent proofs. For details, see [39–41].

Assume that $X(t)$ is a regular time-homogeneous Markov process in the d-dimensional space E_d satisfying the stochastic equation

$$dX(t) = b(X)dt + \sum_{r=1}^{k} g_r(X) dB_r(t).$$

Then the diffusion matrix has the form

$$A(X) = (a_{ij}(x)), \qquad a_{ij}(x) = \sum_{r=1}^{k} g_r^i(x) g_r^j(x).$$

Lemma 2 (see [42, 43]). *The Markov process $X(t)$ has a unique ergodic stationary distribution $\pi(\cdot)$ if there exists a bounded domain $D \subset E_d$ with regular boundary Γ such that*

(H1) *there exists a constant $M > 0$ satisfying*

$$\sum_{i,j=1}^{d} a_{ij}(x)\xi_i\xi_j \geqslant M|\xi|^2, \qquad x \in D,\ \xi \in \mathbb{R}^d;$$

(H2) *there is a C^2-function $V \geqslant 0$ such that LV is negative for any $x \in E_d \setminus D$. Then*

$$P\Big\{\limsup_{T\to\infty} \frac{1}{T}\int_0^T f(X(t))dt = \int_{E_d} f(x)\pi(dx)\Big\} = 1$$

for all $x \in E_d$, where $f(\cdot)$ is a function integrable with respect to the measure π.

Theorem 3. *Assume that*

$$R_0^s := \frac{\beta\mu}{(\mu+\frac{1}{2}\sigma_1^2)(\gamma_1+\mu+\frac{1}{2}\sigma_2^2)} + \frac{\alpha\beta q\gamma_1\mu}{(\mu+\frac{1}{2}\sigma_1^2)(\gamma_1+\mu+\frac{1}{2}\sigma_2^2)(\gamma_2+\mu+\frac{1}{2}\sigma_3^2)} > 1,$$

then system (3) has a unique stationary distribution $\pi(\cdot)$ and the ergodicity holds.

Proof. In order to prove Theorem 3, we need to verify the conditions of Lemma 2. First, we verify that condition (H2) of Lemma 2 is valid. Define

$$V_1 = -\ln I - (a_1+b_1)\ln S - b_2\ln C + \frac{\alpha\beta(a_1+b_1)}{\gamma_2+\mu}C,$$
$$V_2 = -\ln S - \ln C + \frac{\alpha\beta}{\gamma_2+\mu}C,$$
$$V_3 = \frac{1}{\theta+1}(S+I+C)^{\theta+1},$$

where a_1, b_1, b_2 are positive constants to be determined later. $\theta \in (0,1)$ satisfy

$$\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2) > 0.$$

Applying the Itô formula, we obtain

$$\begin{aligned}
LV_1 = -\beta S - \frac{\alpha\beta SC}{I} + \gamma_1 + \mu + \frac{1}{2}\sigma_2^2 - \frac{a_1\mu}{S} - \frac{b_1\mu}{S} + \beta(a_1+b_1)I + \frac{\alpha\beta q\gamma_1(a_1+b_1)}{\gamma_2+\mu}I+\\
+\mu(a_1+b_1) + \frac{1}{2}(a_1+b_1)\sigma_1^2 - \frac{b_2 q\gamma_1 I}{C} + b_2\gamma_2 + b_2\mu + \frac{1}{2}b_2\sigma_3^2 =\\
= -\beta S - \frac{a_1\mu}{S} + a_1(\mu+\frac{1}{2}\sigma_1^2) - \frac{\alpha\beta SC}{I} - \frac{b_1\mu}{S} - \frac{b_2 q\gamma_1 I}{C} + b_1(\mu+\frac{1}{2}\sigma_1^2)+\\
+b_2(\gamma_2+\mu+\frac{1}{2}\sigma_3^2) + \gamma_1 + \mu + \frac{1}{2}\sigma_2^2 + \frac{\beta(a_1+b_1)(\gamma_2+\mu+\alpha q\gamma_1)}{\gamma_2+\mu}I \leqslant\\
\leqslant -2\sqrt{\beta a_1\mu} + a_1(\mu+\frac{1}{2}\sigma_1^2) - 3\sqrt[3]{\alpha\beta q\gamma_1\mu b_1 b_2} + b_1(\mu+\frac{1}{2}\sigma_1^2)+\\
+b_2(\gamma_2+\mu+\frac{1}{2}\sigma_3^2) + \gamma_1 + \mu + \frac{1}{2}\sigma_2^2 + \frac{\beta(a_1+b_1)(\gamma_2+\mu+\alpha q\gamma_1)}{\gamma_2+\mu}I.
\end{aligned}$$

Let

$$a_1 = \frac{\beta\mu}{(\mu+\frac{1}{2}\sigma_1^2)^2}, \quad b_1 = \frac{\alpha\beta q\gamma_1\mu}{(\mu+\frac{1}{2}\sigma_1^2)^2(\gamma_2+\mu+\frac{1}{2}\sigma_3^2)}, \quad b_2 = \frac{\alpha\beta q\gamma_1\mu}{(\mu+\frac{1}{2}\sigma_1^2)(\gamma_2+\mu+\frac{1}{2}\sigma_3^2)^2}.$$

Then we have

$$\begin{aligned} LV_1 \leqslant & -\frac{\beta\mu}{\mu+\frac{1}{2}\sigma_1^2} - \frac{\alpha\beta q\gamma_1\mu}{(\mu+\frac{1}{2}\sigma_1^2)(\gamma_2+\mu+\frac{1}{2}\sigma_3^2)}+ \\ & +\gamma_1+\mu+\frac{1}{2}\sigma_2^2 + \frac{\beta(a_1+b_1)(\gamma_2+\mu+\alpha q\gamma_1)}{\gamma_2+\mu}I = \\ = & -(\gamma_1+\mu+\frac{1}{2}\sigma_2^2)(R_0^s-1) + \frac{\beta(a_1+b_1)(\gamma_2+\mu+\alpha q\gamma_1)}{\gamma_2+\mu}I =: \\ =: & -\lambda + \frac{\beta(a_1+b_1)(\gamma_2+\mu+\alpha q\gamma_1)}{\gamma_2+\mu}I. \end{aligned} \tag{9}$$

where

$$\lambda = (\gamma_1+\mu+\frac{1}{2}\sigma_2^2)(R_0^s-1) > 0.$$

Also, we have

$$LV_2 = -\frac{\mu}{S} - \frac{q\gamma_1 I}{C} + \frac{\beta(\gamma_2+\mu+\alpha q\gamma_1)}{\gamma_2+\mu}I + \gamma_2 + 2\mu + \frac{\sigma_1^2+\sigma_3^2}{2}. \tag{10}$$

$$\begin{aligned} LV_3 = & (S+I+C)^\theta[\mu-\mu(S+I+C)-(\gamma_1-q\gamma_1)I-\gamma_2 C]+ \\ & +\frac{\theta}{2}(S+I+C)^{\theta-1}(\sigma_1^2S^2+\sigma_2^2I^2+\sigma_3^2C^2) \leqslant \\ \leqslant & \mu(S+I+C)^\theta - \mu(S+I+C)^{\theta+1} + \frac{\theta}{2}(S+I+C)^{\theta-1}(\sigma_1^2S^2+\sigma_2^2I^2+\sigma_3^2C^2) \leqslant \\ \leqslant & \mu(S+I+C)^\theta - [\mu-\frac{\theta}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2)](S+I+C)^{\theta+1} \leqslant \\ \leqslant & B - \frac{1}{2}[\mu-\frac{\theta}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2)](S+I+C)^{\theta+1} \leqslant \\ \leqslant & B - \frac{1}{2}[\mu-\frac{\theta}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2)](S^{\theta+1}+I^{\theta+1}+C^{\theta+1}), \end{aligned} \tag{11}$$

where

$$B = \sup_{(S,I,C)\in\mathbb{R}_+{}^3}\left\{\mu(S+I+C)^\theta - \frac{1}{2}\left[\mu-\frac{\theta}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2)\right](S+I+C)^{\theta+1}\right\} < \infty.$$

Define a C^2-function $\hat{V}(S,I,C):\mathbb{R}_+{}^3\to\mathbb{R}$ as follows:

$$\hat{V}(S,I,C) = MV_1+V_2+V_3,$$

where M is a positive constant to be determined later. It is obvious that

$$\liminf_{k\to\infty,(S,I,C)\in\mathbb{R}_+{}^3\setminus U_k}\hat{V}(S,I,C) = +\infty.$$

where $U_k = (\frac{1}{k},k)\times(\frac{1}{k},k)\times(\frac{1}{k},k)$. So $\hat{V}(S,I,C)$ is a continuous function which possesses a minimum point $(\bar{S}_0,\bar{I}_0,\bar{C}_0)$ in the interior of $\mathbb{R}_+{}^3$.

Therefore, we can define a non-negative C^2-function $V(S,I,C):\mathbb{R}_+{}^3\to\mathbb{R}$ as follows

$$V(S,I,C)=\hat{V}(S,I,C)-\hat{V}(\bar{S}_0,\bar{I}_0,\bar{C}_0).$$

From (9), (10) and (11), we have that

$$\begin{aligned}LV\leqslant & -M\lambda-\frac{\mu}{S}-\frac{q\gamma_1 I}{C}+\frac{\beta(\gamma_2+\mu+\alpha q\gamma_1)(Ma_1+Mb_1+1)}{\gamma_2+\mu}I+\gamma_2+2\mu+\\ &+\frac{\sigma_1^2+\sigma_3^2}{2}+B-\frac{1}{2}[\mu-\frac{\theta}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2)](S^{\theta+1}+I^{\theta+1}+C^{\theta+1})=:\\ &=:-M\lambda-\frac{\mu}{S}-\frac{q\gamma_1 I}{C}+\frac{\beta(\gamma_2+\mu+\alpha q\gamma_1)(Ma_1+Mb_1+1)}{\gamma_2+\mu}I+Q(S,I,C),\end{aligned}$$

where

$$Q(S,I,C)=\gamma_2+2\mu+\frac{\sigma_1^2+\sigma_3^2}{2}+B-\frac{1}{2}[\mu-\frac{\theta}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2)](S^{\theta+1}+I^{\theta+1}+C^{\theta+1}).$$

Let M be a positive constant satisfying

$$-M\lambda+F\leqslant-2, \tag{12}$$

where

$$F=\sup_{(S,I,C)\in\mathbb{R}_+{}^3}Q(S,I,C)<\infty.$$

Now, we define a bounded closed set as

$$D_\varepsilon=\left\{(S,I,C)\in\mathbb{R}_+{}^3:\varepsilon\leqslant S\leqslant\frac{1}{\varepsilon},\varepsilon\leqslant I\leqslant\frac{1}{\varepsilon},\varepsilon^2\leqslant C\leqslant\frac{1}{\varepsilon^2}\right\}.$$

where $\varepsilon>0$ is a sufficiently small positive constant satisfying the following inequalities:

$$-\frac{\mu}{\varepsilon}+U\leqslant-1, \tag{13}$$

$$-M\lambda+\frac{\beta(\gamma_2+\mu+\alpha q\gamma_1)(Ma_1+Mb_1+1)}{\gamma_2+\mu}\varepsilon+F\leqslant-1, \tag{14}$$

$$-\frac{q\gamma_1}{\varepsilon}+U\leqslant-1, \tag{15}$$

$$-\frac{1}{4}\left[\mu-\frac{\theta}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2)\right]\frac{1}{\varepsilon^{\theta+1}}+G\leqslant-1, \tag{16}$$

$$-\frac{1}{4}\left[\mu-\frac{\theta}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2)\right]\frac{1}{\varepsilon^{\theta+1}}+H\leqslant-1, \tag{17}$$

$$-\frac{1}{4}\left[\mu-\frac{\theta}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2)\right]\frac{1}{\varepsilon^{2(\theta+1)}}+N\leqslant-1. \tag{18}$$

Therefore, $\mathbb{R}_+{}^3\setminus D_\varepsilon$ can be divided into six domains:

$$\begin{gathered}D_1=\{(S,I,C)\in\mathbb{R}_+{}^3,0<S<\varepsilon\},\ \ D_2=\{(S,I,C)\in\mathbb{R}_+{}^3,0<I<\varepsilon\},\\ D_3=\{(S,I,C)\in\mathbb{R}_+{}^3,0<C<\varepsilon^2,I\geqslant\varepsilon\},\ \ D_4=\{(S,I,C)\in\mathbb{R}_+{}^3,S>\frac{1}{\varepsilon}\},\\ D_5=\{(S,I,C)\in\mathbb{R}_+{}^3,I>\frac{1}{\varepsilon}\},\ \ D_6=\{(S,I,C)\in\mathbb{R}_+{}^3,C>\frac{1}{\varepsilon^2}\}.\end{gathered}$$

In the following we show that $LV(S,I,C) \leqslant -1$ for any given $(S,I,C) \in {\mathbb{R}_+}^3 \setminus D_\varepsilon$.

Case 1. If $(S,I,C) \in D_1$, then we have that

$$\begin{gathered} LV \leqslant -M\lambda - \frac{\mu}{S} - \frac{q\gamma_1 I}{C} + \\ + \frac{\beta(\gamma_2+\mu+\alpha q\gamma_1)(Ma_1+Mb_1+1)}{\gamma_2+\mu} I + Q(S,I,C) \leqslant -\frac{\mu}{\varepsilon} + U. \end{gathered} \tag{19}$$

where

$$U = \sup_{(S,I,C)\in {\mathbb{R}_+}^3} \left\{ \frac{\beta(\gamma_2+\mu+\alpha q\gamma_1)(Ma_1+Mb_1+1)}{\gamma_2+\mu} I + Q(S,I,C) \right\} < \infty. \tag{20}$$

It is easy for us to see that $LV(S,I,C) \leqslant -1$ for any given $(S,I,C) \in D_1$ by means of (13).

Case 2. If $(S,I,C) \in D_2$, then we get that

$$\begin{aligned} LV &\leqslant -M\lambda - \frac{\mu}{S} + \frac{\beta(\gamma_2+\mu+\alpha q\gamma_1)(Ma_1+Mb_1+1)}{\gamma_2+\mu} I + Q(S,I,C) \leqslant \\ &\leqslant -M\lambda + \frac{\beta(\gamma_2+\mu+\alpha q\gamma_1)(Ma_1+Mb_1+1)}{\gamma_2+\mu} I + F \leqslant \\ &\leqslant -M\lambda + \frac{\beta(\gamma_2+\mu+\alpha q\gamma_1)(Ma_1+Mb_1+1)}{\gamma_2+\mu} \varepsilon + F. \end{aligned} \tag{21}$$

According to (14), the conclusion that $LV(S,I,C) \leqslant -1$ for any given $(S,I,C) \in D_2$ is true.

Case 3. If $(S,I,C) \in D_3$, then we see that

$$LV \leqslant -\frac{q\gamma_1 I}{C} + \frac{\beta(\gamma_2+\mu+\alpha q\gamma_1)(Ma_1+Mb_1+1)}{\gamma_2+\mu} I + Q(S,I,C) \leqslant -\frac{q\gamma_1}{C} + U. \tag{22}$$

By means of (15), we obtain the conclusion that $LV(S,I,C) \leqslant -1$ for any given $(S,I,C) \in D_3$.

Case 4. If $(S,I,C) \in D_4$, then we attain that

$$\begin{gathered} LV \leqslant -\frac{1}{4}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right] S^{\theta+1} - \\ -\frac{1}{4}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right] S^{\theta+1} + \gamma_2 + 2\mu + \frac{\sigma_1^2+\sigma_3^2}{2} + \\ + \frac{\beta(\gamma_2+\mu+\alpha q\gamma_1)(Ma_1+Mb_1+1)}{\gamma_2+\mu} I + B - \frac{1}{2}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right] (I^{\theta+1} + C^{\theta+1}) \leqslant \\ \leqslant -\frac{1}{4}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right] S^{\theta+1} + G \leqslant -\frac{1}{4}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right] \frac{1}{\varepsilon^{\theta+1}} + G. \end{gathered} \tag{23}$$

where

$$\begin{gathered} G = \sup_{(S,I,C)\in {\mathbb{R}_+}^3} \Big\{ -\frac{1}{4}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right] S^{\theta+1} + \gamma_2 + 2\mu + \frac{\sigma_1^2+\sigma_3^2}{2} + \\ + \frac{\beta(\gamma_2+\mu+\alpha q\gamma_1)(Ma_1+Mb_1+1)}{\gamma_2+\mu} I + \\ + B - \frac{1}{2}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right] (I^{\theta+1} + C^{\theta+1}) \Big\} < \infty. \end{gathered} \tag{24}$$

It is obvious that $LV(S,I,C) \leqslant -1$ for any given $(S,I,C) \in D_4$ in virtue of (16).

Case 5. If $(S,I,C) \in D_5$, then we arrive at

$$\begin{aligned} LV \leqslant & -\frac{1}{4}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right] I^{\theta+1} - \\ & -\frac{1}{4}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right] I^{\theta+1} + \gamma_2 + 2\mu + \frac{\sigma_1^2+\sigma_3^2}{2} + \\ & + \frac{\beta(\gamma_2+\mu+\alpha q\gamma_1)(Ma_1+Mb_1+1)}{\gamma_2+\mu} I + B - \frac{1}{2}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right](S^{\theta+1}+C^{\theta+1}) \leqslant \\ & \leqslant -\frac{1}{4}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right] I^{\theta+1} + H \leqslant -\frac{1}{4}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right]\frac{1}{\varepsilon^{\theta+1}} + H. \end{aligned} \tag{25}$$

where

$$\begin{aligned} H = \sup_{(S,I,C)\in\mathbb{R}_+{}^3} \Big\{ & -\frac{1}{4}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right] I^{\theta+1} + \gamma_2 + 2\mu + \frac{\sigma_1^2+\sigma_3^2}{2} + \\ & + \frac{\beta(\gamma_2+\mu+\alpha q\gamma_1)(Ma_1+Mb_1+1)}{\gamma_2+\mu} I + \\ & + B - \frac{1}{2}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right](S^{\theta+1}+C^{\theta+1}) \Big\} < \infty. \end{aligned} \tag{26}$$

With the aid of (17), the result that $LV(S,I,C) \leqslant -1$ for any given $(S,I,C) \in D_5$ is right.

Case 6. If $(S,I,C) \in D_6$, then we have a conclusion as follows

$$\begin{aligned} LV \leqslant & -\frac{1}{4}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right] C^{\theta+1} - \\ & -\frac{1}{4}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right] C^{\theta+1} + \gamma_2 + 2\mu + \frac{\sigma_1^2+\sigma_3^2}{2} + \\ & + \frac{\beta(\gamma_2+\mu+\alpha q\gamma_1)(Ma_1+Mb_1+1)}{\gamma_2+\mu} I + B - \frac{1}{2}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right](S^{\theta+1}+I^{\theta+1}) \leqslant \\ & \leqslant -\frac{1}{4}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right] C^{\theta+1} + N \leqslant -\frac{1}{4}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right]\frac{1}{\varepsilon^{2(\theta+1)}} + N. \end{aligned} \tag{27}$$

where

$$\begin{aligned} N = \sup_{(S,I,C)\in\mathbb{R}_+{}^3} \Big\{ & -\frac{1}{4}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right] C^{\theta+1} + \gamma_2 + 2\mu + \frac{\sigma_1^2+\sigma_3^2}{2} + \\ & + \frac{\beta(\gamma_2+\mu+\alpha q\gamma_1)(Ma_1+Mb_1+1)}{\gamma_2+\mu} I + \\ & + B - \frac{1}{2}\left[\mu - \frac{\theta}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right](S^{\theta+1}+I^{\theta+1}) \Big\} < \infty. \end{aligned} \tag{28}$$

where U, G, H, N are all positive constants defined by (20), (24), (26), (28). We draw the conclusion that $LV(S,I,C) \leqslant -1$ for any given $(S,I,C) \in D_6$ by virtue of (18).

From the discussion of a series of cases above, we come to a conclusion that

$$LV(S,I,C) \leqslant -1, \quad (S,I,C) \in \mathbb{R}_+{}^3 \setminus D_\varepsilon.$$

Next, the diffusion matrix of system (3) is

$$A = \begin{pmatrix} \sigma_1^2 S^2 & 0 & 0 \\ 0 & \sigma_2^2 I^2 & 0 \\ 0 & 0 & \sigma_3^2 C^2 \end{pmatrix}.$$

Let $c = \min_{(S,I,C)\in \bar{D}_k \subset \mathbb{R}_+{}^3} \{\sigma_1^2 S^2, \sigma_2^2 I^2, \sigma_3^2 C^2\}$, then we can have

$$\sum_{i,j=1}^{3} a_{ij}(x)\xi_i\xi_j = \sigma_1^2 S^2 \xi_1^2 + \sigma_2^2 I^2 \xi_2^2 + \sigma_3^2 C^2 \xi_3^2 \geqslant c|\xi|^2, \quad (S,I,C) \in \bar{D}_k, \xi \in \mathbb{R}^3.$$

Hence condition (H1) of Lemma 2 is verified.

As a result, according to Lemma 2, there exists a unique ergodic stationary distribution. The proof is finished.

4 Extinction and persistence of system (3)

For convenience, we introduce the notation $\langle f(t)\rangle = \frac{1}{t}\int_0^t f(\varsigma)d\varsigma$, where $f(t)$ is an integrable function on $[0,+\infty)$, and the following lemmas.

Lemma 4 (Strong law of large numbers, see [38])**.** *Let $M = \{M_t\}_{t\geqslant 0}$ be a real-valued continuous local martingale vanishing at $t = 0$. Then*

$$\lim_{t\to\infty} \langle M, M\rangle_t = \infty \quad \text{a.s.} \;\Rightarrow\; \lim_{t\to\infty} \frac{M_t}{\langle M, M\rangle_t} = 0 \quad \text{a.s.}$$

Also

$$\limsup_{t\to\infty} \frac{\langle M, M\rangle_t}{t} < \infty \quad \text{a.s.} \;\Rightarrow\; \lim_{t\to\infty} \frac{M_t}{t} = 0 \quad \text{a.s.}$$

Lemma 5. *Let $(S(t), I(t), C(t))$ be the solution of system (3) with the initial value $(S(0), I(0), C(0)) \in \mathbb{R}_+{}^3$. Then*

$$\limsup_{t\to\infty}(S(t) + I(t) + C(t)) < \infty \quad \text{a.s.}$$

as well as

$$\lim_{t\to\infty} \frac{S(t)}{t} = 0, \lim_{t\to\infty} \frac{I(t)}{t} = 0, \lim_{t\to\infty} \frac{C(t)}{t} = 0, \;\text{a.s.} \tag{29}$$

$$\lim_{t\to\infty} \frac{\ln S(t)}{t} = 0, \lim_{t\to\infty} \frac{\ln I(t)}{t} = 0, \lim_{t\to\infty} \frac{\ln C(t)}{t} = 0, \;\text{a.s.} \tag{30}$$

and

$$\lim_{t\to\infty} \frac{\int_0^t S(\varsigma)dB_1(\varsigma)}{t} = 0, \lim_{t\to\infty} \frac{\int_0^t I(\varsigma)dB_2(\varsigma)}{t} = 0, \lim_{t\to\infty} \frac{\int_0^t C(\varsigma)dB_3(\varsigma)}{t} = 0, \;\text{a.s.} \tag{31}$$

Proof. From system (3) we have

$$\begin{aligned} d(S+I+C) = [\mu - \mu(S+I+C) - (1-q)\gamma_1 I - \gamma_2 C]dt + \\ +\sigma_1 S dB_1(t) + \sigma_2 I dB_2(t) + \sigma_3 C dB_3(t). \end{aligned} \tag{32}$$

By solving equation (32), we can get

$$\begin{aligned} S(t)+I(t)+C(t) = 1 + (S(0)+I(0)+C(0)-1)e^{-\mu t} - (1-q)\gamma_1 \int_0^t e^{-\mu(t-\varsigma)} I(\varsigma) d\varsigma - \\ -\gamma_2 \int_0^t e^{-\mu(t-\varsigma)} C(\varsigma) d\varsigma + \sigma_1 \int_0^t e^{-\mu(t-\varsigma)} S(\varsigma) dB_1(\varsigma) + \\ +\sigma_2 \int_0^t e^{-\mu(t-\varsigma)} I(\varsigma) dB_2(\varsigma) + \sigma_3 \int_0^t e^{-\mu(t-\varsigma)} C(\varsigma) dB_3(\varsigma) \leqslant \\ \leqslant 1 + (S(0)+I(0)+C(0)-1)e^{-\mu t} + N(t) \quad \text{a.s.}, \end{aligned} \tag{33}$$

where

$$N(t) = \sigma_1 \int_0^t e^{-\mu(t-\varsigma)} S(\varsigma) dB_1(\varsigma) + \sigma_2 \int_0^t e^{-\mu(t-\varsigma)} I(\varsigma) dB_2(\varsigma) + \sigma_3 \int_0^t e^{-\mu(t-\varsigma)} C(\varsigma) dB_3(\varsigma).$$

It is apparent that $N(t)$ is a continuous local martingale having the property of $N(0)=0$. Let

$$X(t) = X(0) + W(t) - P(t) + N(t),$$

where

$$X(0) = S(0)+I(0)+C(0), W(t) = 1-e^{-\mu t}, \ P(t) = [S(0)+I(0)+C(0)](1-e^{-\mu t}). \tag{34}$$

By means of (34), we can get that

$$S(t)+I(t)+C(t) \leqslant X(t) \quad \text{a.s.}, \quad t \geqslant 0.$$

From the expressions of $W(t)$ and $P(t)$, it is easy to see that $W(t)$ and $P(t)$ are continuous increasing processes with $W(0)=P(0)=0$. So, according to Theorem 3.9 in [44, 45], we obtain $\lim\limits_{t\to\infty} X(t) < \infty$ a.s. Therefore

$$\limsup_{t\to\infty} (S(t)+I(t)+C(t)) < \infty \quad \text{a.s.}$$

Let

$$N_1(t) = \int_0^t S(\varsigma) dB_1(\varsigma), N_2(t) = \int_0^t I(\varsigma) dB_2(\varsigma), N_3(t) = \int_0^t C(\varsigma) dB_3(\varsigma).$$

Then we can obtain

$$\limsup_{t\to\infty} \frac{\langle N_1, N_1 \rangle_t}{t} \leqslant \sup_{t\geqslant 0} S^2(t) < \infty.$$

So, according to Lemma 4, we have $\lim\limits_{t\to\infty} \dfrac{\int_0^t S(\varsigma) dB_1(\varsigma)}{t} = 0$ a.s. In a similar way, we obtain

$$\lim_{t\to\infty} \frac{\int_0^t I(\varsigma) dB_2(\varsigma)}{t} = 0, \quad \lim_{t\to\infty} \frac{\int_0^t C(\varsigma) dB_3(\varsigma)}{t} = 0 \quad \text{a.s.}$$

Next, because $S(t), I(t), C(t)$ are bounded and positive, we have

$$\lim_{t\to\infty} \frac{S(t)}{t} = 0, \quad \lim_{t\to\infty} \frac{I(t)}{t} = 0, \quad \lim_{t\to\infty} \frac{C(t)}{t} = 0 \quad \text{a.s.}$$

and

$$\lim_{t\to\infty} \frac{\ln S(t)}{t} = 0, \quad \lim_{t\to\infty} \frac{\ln I(t)}{t} = 0, \quad \lim_{t\to\infty} \frac{\ln C(t)}{t} = 0 \quad \text{a.s.}$$

This finishes the proof.

4.1 Disease extinction

In this subsection, we derive in detail the condition for the extinction of the disease.

Theorem 6. *Assume that* $(S(t), I(t), C(t))$ *is the solution of the system* (3) *having the initial value* $(S(0), I(0), C(0)) \in \mathbb{R}_+{}^3$. *If*

$$\bar{R}_0 = \frac{6\beta(\gamma_1+\mu)(\gamma_2+\mu+\alpha q\gamma_1)}{(\gamma_1+\mu)^2(\gamma_2+\mu+\frac{\sigma_3^2}{2}) \wedge \frac{\sigma_2^2 q^2\gamma_1^2}{2}} < 1,$$

then

$$\limsup_{t\to\infty} \frac{\ln[q\gamma_1 I + (\gamma_1+\mu)C]}{t} \leqslant \frac{(\gamma_1+\mu)^2(\gamma_2+\mu+\frac{\sigma_3^2}{2}) \wedge \frac{\sigma_2^2 q^2\gamma_1^2}{2}}{2(\gamma_1+\mu)^2}(\bar{R}_0 - 1) < 0$$

as well as

$$\limsup_{t\to\infty}\langle S(t)\rangle = 1 \quad a.s.$$

Proof. Let $V_0(t) = \ln[q\gamma_1 I(t) + (\gamma_1+\mu)C(t)]$; applying the Itô formula to $V_0(t)$, we have

$$\begin{aligned}
dV_0(t) = \Big\{&\frac{q\gamma_1\beta SI + q\gamma_1\alpha\beta SC - (\gamma_1+\mu)(\gamma_2+\mu)C}{q\gamma_1 I + (\gamma_1+\mu)C} - \frac{q^2\gamma_1^2\sigma_2^2 I^2 + (\gamma_1+\mu)^2\sigma_3^2 C^2}{2[q\gamma_1 I + (\gamma_1+\mu)C]^2}\Big\}dt+\\
&+\frac{\sigma_2 q\gamma_1 I}{q\gamma_1 I + (\gamma_1+\mu)C}dB_2(t) + \frac{\sigma_3(\gamma_1+\mu)C}{q\gamma_1 I + (\gamma_1+\mu)C}dB_3(t) \leqslant\\
\leqslant \frac{\beta(\gamma_1+\mu+\alpha q\gamma_1)S}{\gamma_1+\mu}dt &- \Big[\frac{(\gamma_1+\mu)(\gamma_2+\mu)C}{q\gamma_1 I + (\gamma_1+\mu)C} + \frac{q^2\gamma_1^2\sigma_2^2 I^2 + (\gamma_1+\mu)^2\sigma_3^2 C^2}{2[q\gamma_1 I + (\gamma_1+\mu)C]^2}\Big]dt+\\
&+\frac{\sigma_2 q\gamma_1 I}{q\gamma_1 I + (\gamma_1+\mu)C}dB_2(t) + \frac{\sigma_3(\gamma_1+\mu)C}{q\gamma_1 I + (\gamma_1+\mu)C}dB_3(t) \leqslant\\
\leqslant \frac{\beta(\gamma_1+\mu+\alpha q\gamma_1)S}{\gamma_1+\mu}dt &- \frac{[(\gamma_1+\mu)^2(\gamma_2+\mu+\frac{\sigma_3^2}{2}) \wedge \frac{\sigma_2^2 q^2\gamma_1^2}{2}](I^2+C^2)}{[q\gamma_1 I + (\gamma_1+\mu)C]^2}dt+\\
&+\frac{\sigma_2 q\gamma_1 I}{q\gamma_1 I + (\gamma_1+\mu)C}dB_2(t) + \frac{\sigma_3(\gamma_1+\mu)C}{q\gamma_1 I + (\gamma_1+\mu)C}dB_3(t) \leqslant\\
\leqslant \frac{\beta(\gamma_1+\mu+\alpha q\gamma_1)S}{\gamma_1+\mu}dt &- \frac{1}{2(\gamma_1+\mu)^2}\Big[(\gamma_1+\mu)^2(\gamma_2+\mu+\frac{\sigma_3^2}{2}) \wedge \frac{\sigma_2^2 q^2\gamma_1^2}{2}\Big]dt+\\
&+\frac{\sigma_2 q\gamma_1 I}{q\gamma_1 I + (\gamma_1+\mu)C}dB_2(t) + \frac{\sigma_3(\gamma_1+\mu)C}{q\gamma_1 I + (\gamma_1+\mu)C}dB_3(t).
\end{aligned} \tag{35}$$

In addition, according to system (3), we get

$$\begin{aligned}
d(S+I+C) &= [\mu - \mu(S+I+C) - (\gamma_1 - q\gamma_1)I - \gamma_2 C]dt+\\
&+\sigma_1 S dB_1(t) + \sigma_2 I dB_2(t) + \sigma_3 C dB_3(t) \leqslant\\
&\leqslant [\mu - \mu(S+I+C)]dt + \sigma_1 S dB_1(t) + \sigma_2 I dB_2(t) + \sigma_3 C dB_3(t).
\end{aligned} \tag{36}$$

Integrating both sides of (36) from 0 to t and dividing by t, then taking the limit superior of both sides of (36), and finally using (33) of Lemma 5, we obtain

$$\limsup_{t\to\infty}\langle S(t) + I(t) + C(t)\rangle \leqslant 1 \quad \text{a.s.}$$

Therefore, from this inequality we conclude that

$$\limsup_{t\to\infty}\langle S(t)\rangle \leqslant 1 \quad \text{a.s.} \tag{37}$$

Integrating both sides of (35) from 0 to t and dividing by t, then taking the limit superior of both sides, and using (37) together with the assumption $\bar{R}_0 < 1$, we conclude that

$$\begin{aligned}\limsup_{t\to\infty}\frac{\ln[q\gamma_1 I + (\gamma_1+\mu)C]}{t} \leqslant\\ \leqslant \frac{\beta(\gamma_1+\mu+\alpha q\gamma_1)}{\gamma_1+\mu} - \frac{1}{2(\gamma_1+\mu)^2}[(\gamma_1+\mu)^2(\gamma_2+\mu+\frac{\sigma_3^2}{2})\wedge\frac{\sigma_2^2 q^2\gamma_1^2}{2}] =\\ = \frac{(\gamma_1+\mu)^2(\gamma_2+\mu+\frac{\sigma_3^2}{2})\wedge\frac{\sigma_2^2q^2\gamma_1^2}{2}}{2(\gamma_1+\mu)^2}(\bar{R}_0-1) < 0.\end{aligned} \tag{38}$$

From (38), it is obvious that

$$\lim_{t\to\infty} I(t) = 0, \quad \lim_{t\to\infty} C(t) = 0 \quad \text{a.s.} \tag{39}$$

Next, integrating both sides of system (3), we obtain

$$\begin{aligned}\frac{S(t)-S(0)}{t} &= \mu - \beta\langle S(t)I(t)\rangle - \alpha\beta\langle S(t)C(t)\rangle - \mu\langle S(t)\rangle + \sigma_1\frac{\int_0^t S(\varsigma)dB_1(\varsigma)}{t},\\ \frac{I(t)-I(0)}{t} &= \beta\langle S(t)I(t)\rangle + \alpha\beta\langle S(t)C(t)\rangle - (\gamma_1+\mu)\langle I(t)\rangle + \sigma_2\frac{\int_0^t I(\varsigma)dB_2(\varsigma)}{t},\\ \frac{C(t)-C(0)}{t} &= q\gamma_1\langle I(t)\rangle - (\gamma_2+\mu)\langle C(t)\rangle + \sigma_3\frac{\int_0^t C(\varsigma)dB_3(\varsigma)}{t}.\end{aligned} \tag{40}$$

Adding up the three equalities of (40) yields

$$\begin{aligned}\frac{S(t)-S(0)}{t} + \frac{I(t)-I(0)}{t} + \frac{C(t)-C(0)}{t} =\\ = \mu - \mu\langle S(t)\rangle - (\gamma_1+\mu-q\gamma_1)\langle I(t)\rangle - (\gamma_2+\mu)\langle C(t)\rangle +\\ +\sigma_1\frac{\int_0^t S(\varsigma)dB_1(\varsigma)}{t} + \sigma_2\frac{\int_0^t I(\varsigma)dB_2(\varsigma)}{t} + \sigma_3\frac{\int_0^t C(\varsigma)dB_3(\varsigma)}{t}.\end{aligned} \tag{41}$$

By (29), (31), (39), (41), we draw the conclusion

$$\limsup_{t\to\infty}\langle S(t)\rangle = 1 \quad \text{a.s.}$$

The proof is finished.

4.2 Persistence in mean

In this subsection, we derive the condition for the persistence in mean of the disease; first, we introduce the definition of persistence in mean.

Definition 1. *System* (3) *is said to be persistent in mean if*

$$\lim_{t\to\infty}\frac{1}{t}\int_0^t I(\varsigma)d\varsigma > 0 \quad a.s.$$

Theorem 7. *For a given initial value $(S(0), I(0), C(0)) \in \Gamma$, if*

$$R_0^s := \frac{\beta\mu}{(\mu+\frac{1}{2}\sigma_1^2)(\gamma_1+\mu+\frac{1}{2}\sigma_2^2)} + \frac{\alpha\beta q\gamma_1\mu}{(\mu+\frac{1}{2}\sigma_1^2)(\gamma_1+\mu+\frac{1}{2}\sigma_2^2)(\gamma_2+\mu+\frac{1}{2}\sigma_3^2)} > 1,$$

then

$$\liminf_{t\to\infty}\langle I(t)\rangle \geqslant \frac{(\gamma_2+\mu)(\gamma_1+\mu+\frac{1}{2}\sigma_2^2)(R_0^s-1)}{\beta(a_1+b_1)(\gamma_2+\mu+\alpha q\gamma_1)} \quad a.s.,$$

where

$$a_1 = \frac{\beta\mu}{(\mu+\frac{1}{2}\sigma_1^2)^2}, \quad b_1 = \frac{\alpha\beta q\gamma_1\mu}{(\mu+\frac{1}{2}\sigma_1^2)^2(\gamma_2+\mu+\frac{1}{2}\sigma_3^2)}.$$

In other words, the disease will prevail if $R_0^s > 1$.

Proof. Using the same function as in the proof of Theorem 3, let

$$V_1(S,I,C) = -\ln I - (a_1+b_1)\ln S - b_2\ln C + \frac{\alpha\beta(a_1+b_1)}{\gamma_2+\mu}C,$$

where a_1, b_1, b_2 are the same as in the proof of Theorem 3. Applying the Itô formula to V_1, we obtain

$$dV_1 = LV_1dt - \sigma_2 dB_2(t) - (a_1+b_1)\sigma_1 dB_1(t) - b_2\sigma_3 dB_3(t) + \frac{\alpha\beta(a_1+b_1)}{\gamma_2+\mu}\sigma_3 C dB_3(t) \quad (42)$$

From the proof of Theorem 3, we get

$$\begin{aligned} LV_1 \leqslant& -\frac{\beta\mu}{\mu+\frac{1}{2}\sigma_1^2} - \frac{\alpha\beta q\gamma_1\mu}{(\mu+\frac{1}{2}\sigma_1^2)(\gamma_2+\mu+\frac{1}{2}\sigma_3^2)}+ \\ &+\gamma_1+\mu+\frac{1}{2}\sigma_2^2 + \frac{\beta(a_1+b_1)(\gamma_2+\mu+\alpha q\gamma_1)}{\gamma_2+\mu}I = \\ =& -(\gamma_1+\mu+\frac{1}{2}\sigma_2^2)(R_0^s-1) + \frac{\beta(a_1+b_1)(\gamma_2+\mu+\alpha q\gamma_1)}{\gamma_2+\mu}I. \end{aligned} \quad (43)$$

Next, based on (43), integrating both sides of (42) from 0 to t and dividing by t, we obtain

$$\begin{aligned} \frac{V_1(S(t),I(t),C(t)) - V_1(S(0),I(0),C(0))}{t} \leqslant& -(\gamma_1+\mu+\frac{1}{2}\sigma_2^2)(R_0^s-1)+ \\ &+\frac{\beta(a_1+b_1)(\gamma_2+\mu+\alpha q\gamma_1)}{\gamma_2+\mu}\langle I(t)\rangle - \frac{\varphi(t)}{t}. \end{aligned} \quad (44)$$

where

$$\varphi(t) = \sigma_2 dB_2(t) + (a_1+b_1)\sigma_1 dB_1(t) + b_2\sigma_3 dB_3(t) - \frac{\alpha\beta(a_1+b_1)}{\gamma_2+\mu}\sigma_3 C dB_3(t).$$

According to Lemma 4 and (30) of Lemma 5, we see that

$$\limsup_{t\to\infty}\frac{\varphi(t)}{t} = 0 \quad \text{a.s.}$$

Next, using (30), (44) and the assumption $R_0^s > 1$, and taking the limit inferior of both sides of (44), we have

$$\liminf_{t\to\infty}\langle I(t)\rangle \geqslant \frac{(\gamma_2+\mu)(\gamma_1+\mu+\frac{1}{2}\sigma_2^2)(R_0^s-1)}{\beta(a_1+b_1)(\gamma_2+\mu+\alpha q\gamma_1)}.$$

The proof is completed.

5 Conclusion

Based on the transmission characteristics of potato early blight and late blight, this paper constructs a stochastic SICR transmission model with environmental noise and conducts a comprehensive analysis of its dynamic behaviors. The research proves that the model possesses a unique global positive solution, which guarantees its biological rationality. When the parameter $R_0^s > 1$, the model has an ergodic stationary distribution, and the diseases will prevail continuously. When $\bar{R}_0 < 1$, the diseases will die out eventually. This study provides theoretical support for the early warning, prediction, prevention and control of potato early blight and late blight.

This study has some limitations. On the basis of the current results, further in-depth research can be carried out from multiple perspectives in the future. Firstly, time-delay factors can be introduced to explore the effects of pathogen incubation period and disease detection delay on the transmission dynamics. Secondly, a stochastic dynamic model with periodic coefficients can be established to reflect the seasonal variation of field temperature, humidity and other environmental factors, so as to improve the model's practical fitting performance. Finally, combining with conventional agricultural prevention methods, multiple control measures including chemical treatment, planting of disease-resistant varieties and crop rotation can be integrated into the model framework. A stochastic epidemic model embedded with control strategies will provide more direct and comprehensive theoretical references for the precise prevention and scientific management of potato early blight and late blight.

Declarations

Conflicts of interest. The authors declare no conflict of interest.

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ДИНАМИЧЕСКИЙ АНАЛИЗ СТОХАСТИЧЕСКОЙ SICR-МОДЕЛИ ПЕРЕДАЧИ РАННЕГО И ПОЗДНЕГО ФИТОФТОРОЗА КАРТОФЕЛЯ

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Фитофтороз картофеля раннего и позднего периода — разрушительное грибковое заболевание, серьёзно угрожающее производству картофеля, а процесс его передачи существенно зависит от колебаний окружающей среды. Для точного выявления закономерностей эпидемии этого заболевания в работе построена стохастическая SICR-модель передачи фитофтороза картофеля раннего и позднего периода. С помощью динамического анализа, во-первых, доказано существование единственного глобального положительного решения системы, что обеспечивает биологическую обоснованность модели; во-вторых, на основе теории Хасьминского выведены достаточные условия существования эргодического стационарного распределения модели, что проясняет динамический механизм, лежащий в основе устойчивой эпидемии заболевания; в-третьих, установлены пороговые условия вымирания и сохранения заболевания в среднем, что позволяет определить ключевые точки для профилактики и контроля заболевания. Результаты показывают, что при $R_0^s > 1$ модель имеет эргодическое стационарное распределение и заболевание сохраняется, а при $\bar{R}_0 < 1$ заболевание в конечном итоге исчезает. Исследование предоставляет теоретическую основу для раннего предупреждения о риске и точной профилактики и контроля фитофтороза картофеля раннего и позднего периода.

Ключевые слова: фитофтороз картофеля раннего и позднего периода, стохастическая SICR-модель, стационарное распределение, вымирание болезни, сохранение болезни

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