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MIXED-COMPOSITE-TYPE EQUATIONS AS A MODEL OF ANOMALOUS DIFFUSION IN TUMOR TISSUES

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This paper examines a boundary value problem for a degenerate elliptic equation in a planar domain bounded by a line segment and an analytic curve. The primary objective is to construct an explicit analytical solution using the Green's function together with single- and double-layer potential methods. A rigorous proof of the existence and uniqueness of the solution under mixed boundary conditions is presented. The degeneracy on a portion of the boundary introduces significant analytical difficulties, necessitating the use of advanced techniques in the theory of elliptic equations with singular coefficients. Furthermore, the potential applications of the model in biomedical settings, particularly in oncology, are discussed. The equation captures anomalous diffusion processes in tumor tissues, incorporating the spatial heterogeneity of the medium. This makes the model a valuable tool for analyzing the distribution of drugs, oxygen, and other substances in biological structures characterized by pronounced anisotropy and heterogeneity.

Keywords: Green's function, potential, Jordan arc, mixed type equation, Lipschitz condition, biomedicine, oncology, tumor, anisotropy.

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1 Introduction

Fundamental studies on model equations of composite and mixed–composite types were carried out by A.V. Bitsadze and M.S. Salakhitdinov [1], in the early 1960s. Their works laid the foundation for the formulation of correct boundary value problems for third-order elliptic–hyperbolic and parabolic–hyperbolic equations, in which the principal part of the operator contains derivatives with respect to both spatial and temporal variables.

In [3], a method was proposed for solving boundary value problems for third-order equations of parabolic–hyperbolic and elliptic–hyperbolic types [7–9], by reducing the original problem to an inverse problem for a second-order mixed-type equation with unknown right-hand sides [10, 11]. This approach made it possible to deepen the analysis of the structure of solutions and provided new tools for studying equations of changing type.

In recent years, fractional calculus has become an important tool in the study of mixed-type equations with identical or varying orders of degeneracy. A number of works employ differential operators of both integer [2, 4] and fractional order [6] to investigate qualitative properties of solutions, including regularity and stability. We also note [5], where boundary value problems for loaded third-order parabolic–hyperbolic equations are examined in various types of domains.

1.1 Relevance of the Study

Despite significant progress in the theory of mixed-type equations, many questions related to the correctness and construction of explicit solutions for degenerate elliptic equations in domains of complex geometry remain open. Particularly relevant are problems in which the degeneracy occurs on a portion of the boundary, greatly complicating the analytical treatment. The development of rigorous methods for constructing the solution and analyzing its properties under such conditions is of considerable importance for both the advancement of the theory of degenerate operators and numerous applied problems.

1.2 Applications and Practical Significance

The equation considered in this work belongs to a class of models describing processes of anomalous diffusion in heterogeneous media, including:

- transport of substances in biological tissues,
- diffusion of pharmaceutical agents in tumor structures,
- distribution of oxygen and metabolites in media with pronounced anisotropy,
- transport phenomena in porous and composite materials.

Particular interest arises in biomedical and oncological applications, where degenerate operator structures naturally appear when modeling regions with low permeability, necrotic zones, and sharp variations in tissue density. The construction of an analytical solution based on the Green's function and potential methods provides an opportunity to obtain precise estimates of the solution's behavior near degeneracy regions, making the results of this work highly significant for the analysis of complex biophysical processes.

2 Statement of Problem A

Consider the equation

$$\frac{\partial}{\partial x}\left(y^m u_{xx} + u_{yy}\right) = 0. \tag{1}$$

Let D be a simply connected mixed-type domain in the (x, y)–plane, bounded by a simple Jordan curve σ with endpoints $A(-1, 0)$ and $B(1, 0)$ lying in the upper half-plane $y > 0$, and by the characteristics

$$AC: \quad x - \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = -1,$$

$$BC: \quad x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = 1,$$

of equation (1), emanating from point

$$C = \left(0, -\left(\frac{m+2}{2}\right)^{\frac{2}{m+2}}\right).$$

We assume that the boundary curve σ satisfies the following conditions:

1. The functions $x = x(s)$, $y = y(s)$ defining the parametric representation of σ are continuous together with their first and second derivatives, and their second derivatives satisfy the Holder condition on the interval $0 \leqslant s \leqslant l$, where s is the arc length measured from B to A, and l is the total length of σ.

2. The curve σ terminates with arbitrarily small arcs BB' and AA' of the normal contour

$$x^2 + \frac{4}{(m+2)^2}\, y^{m+2} = 1, \qquad y \geqslant 0.$$

3. Each horizontal line $y = c$, $0 \leqslant c < h$, intersects σ in exactly two points, while the line $y = h$ has a single point of tangency $N(0, h)$ with σ.

We denote the elliptic part of the mixed domain D by D_1 and the hyperbolic part by D_2.

3 Problem A.

Find a regular solution $u(x, y)$ of equation (1) in the elliptic subdomain D_1, satisfying the boundary conditions

$$u|_{\sigma} = f(s), \qquad u|_{AB} = \tau(x), \qquad \left.\frac{\partial u}{\partial n}\right|_{\tau_1} = \varphi(s),$$

where τ_1 denotes the part AN of the curve σ, n is the inward normal to σ, and the given functions $f'(s)$ and $\varphi(s)$ satisfy the Holder condition, while $\tau(x)$ is a Lipschitz continuous function.

To study this problem, we use the representation

$$u(x, y) = z(x, y) + \omega(y), \tag{2}$$

where $z(x, y)$ and $\omega(y)$ are solutions of equation (1). Throughout the arc τ_1 (except for its endpoint N) we assume that the geometric condition

$$\frac{dx}{dn} \neq 0, \tag{3}$$

holds.

3.1 Uniqueness of the solution to Problem A

The uniqueness of the solution to Problem A under condition (4) is readily established. Without loss of generality, in the representation (3) we may choose the function $\omega(y)$ so that

$$\omega(0) = \omega(h) = 0. \tag{4}$$

Then the homogeneous problem is reduced to finding in the domain D_1 a regular solution $z(x, y)$ of the equation

$$y^m z_{xx} + z_{yy} = 0, \tag{5}$$

satisfying the boundary conditions

$$z|_{\tau_1} = -\,\omega(y), \qquad z|_{AB} = 0, \qquad \left.\frac{\partial z}{\partial n}\right|_{\sigma_1} = -\,\omega'(y)\,\frac{dy}{dn}. \tag{6}$$

A regular solution of equation (6) cannot attain a positive maximum or a negative minimum in the interior of D_1. By virtue of (5) and the second condition of (7), the function $z(x, y)$ cannot attain a non-zero extremum at the point N nor on the segment AB.

It also cannot attain a non-zero extremum at any point of the open arc σ_1. Indeed, suppose that z attains an extremum at some point of σ_1. Then, by the first condition in (7),

$$\frac{\partial z}{\partial s} = \omega'(y)\,\frac{dx}{dn} = 0.$$

Since condition (4) ensures that $\frac{dx}{dn} \neq 0$ on σ_1, we obtain $\omega'(y) = 0$. Hence, using the third condition in (7), we find that

$$\frac{\partial z}{\partial n} = 0.$$

at the same boundary point. This contradicts the classical boundary point maximum principle for elliptic equations (see, e.g.), [1].

Thus $z(x, y)$ has no non-zero extremum on σ_1. Finally, by condition (2), it also cannot attain a non-zero extremum on the arc BN. Therefore, the only solution of the homogeneous problem is the trivial one, which completes the proof of uniqueness for Problem A.

3.2 Existence of the solution to problem A.

Without loss of generality, we assume that

$$u(A) = u(B) = u'(A) = u'(B) = 0.$$

Using representation (3), conditions (5) and boundary data (2), the solution of problem A reduces to finding in the domain D_1 a regular solution of equation (6) satisfying the boundary conditions

$$z|_{\tau} = f(s) - \omega[y(s)], \quad z|_{AB} = \tau(x), \tag{7}$$

$$\left.\frac{\partial z}{\partial n}\right|_{\sigma_1} = \varphi(s) - \omega'(y)\cos(n, y). \tag{8}$$

In what follows, we will need an explicit representation of the solution to problem (6),(7). To obtain it, we use the Green function. According to Gellerstedt, the Green function of problem (6) can be constructed by the potential method [2, 3].

Equation (6) admits the fundamental solution

$$g(x, y; x_0, y_0) = k\Big(\tfrac{4}{m+2}\Big)^{4\beta-2} (r_1^2)^{-\beta}\Big(1 - \tfrac{r^2}{r_1^2}\Big)^{1-2\beta} F\Big(1-\beta, 1-\beta, 2-2\beta,\, 1 - \tfrac{r^2}{r_1^2}\Big),$$

where

$$\begin{cases} r^2 = (x-x_0)^2 + \dfrac{4}{(m+2)^2}\Big(y^{\frac{m+2}{2}} - y_0^{\frac{m+2}{2}}\Big)^2, \\[2mm] r_1^2 = (x-x_0)^2 + \dfrac{4}{(m+2)^2}\Big(y^{\frac{m+2}{2}} + y_0^{\frac{m+2}{2}}\Big)^2, \end{cases}$$

$$k = \frac{1}{4\pi}\Big(\tfrac{4}{m+2}\Big)^{2-2\beta}\frac{\Gamma(1-\beta)^2}{\Gamma(2-2\beta)}, \quad \beta = \frac{m}{2(m+2)}.$$

The function $g(x, y; x_0, y_0)$ has a logarithmic singularity and satisfies

$$g(x, 0; x_0, y_0) = 0, \quad y_0 > 0,\ x \in \mathbb{R}.$$

Introduce the operator

$$A_s\left[g\left(\xi,\eta;x,y\right)\right]=\eta^m\frac{d\eta}{\partial s}\cdot\frac{\partial g}{\partial\xi}-\frac{d\xi}{ds}\cdot\frac{\partial g}{\partial\eta}, \tag{9}$$

here (ξ,η) - denotes the coordinates of the variable point on the curve σ (9). Through direct calculation, we find that

$$\begin{gathered}A_s\left[g\left(\xi,\eta;x,y\right)\right]=\frac{m}{4}\eta'\xi'\left(s\right)g\left(\xi,\eta;x,y\right)-\\ -\frac{k\left(1-\beta\right)y}{\left(r_1{}^2\right)^{1-\beta}r^2}F\left(1-\beta,-\beta,2-2\beta,1-\frac{r^2}{r_1{}^2}\right)\times\\ \times\left[2\eta^{m+1}\eta'\left(s\right)\left(\xi-x\right)+\frac{m+2}{2}\xi'\left(s\right)\left(\xi-x\right)^2+\frac{2\xi'\left(s\right)}{m+2}\left(y^{m+2}-\eta^{m+2}\right)\right].\end{gathered}$$

Because of the smoothness assumptions on σ (10),

$$\int_0^l\left|A_s[g]\right|\,ds\leqslant C,$$

where the constant C does not depend on (x,y).

It is known (see [2, 4]) that

$$\int_0^l A_s[g(\xi,\eta;x,y)]\,ds=\begin{cases}i(x,y)-1, & (x,y)\in D_1,\\ i(x,y)-\frac{1}{2}, & (x,y)\in\sigma,\\ i(x,y), & (x,y)\notin\overline{D_1},\end{cases} \tag{10}$$

where

$$i(x,y)=\int_{-1}^{1}\frac{\partial g(\xi,0;x,y)}{\partial\eta}\,d\xi.$$

Consider the double-layer potential

$$\omega(x,y)=\int_0^l\mu(t)\,A_t[g(\xi(t),\ \ \eta(t);x,y)]\,dt, \tag{11}$$

where $\mu\in C[0,l]$. The jump relations are

$$\begin{cases}\omega_i(s)=-\frac{1}{2}\mu(s)+\omega_0(s),\\ \omega_e(s)=\frac{1}{2}\mu(s)+\omega_0(s),\end{cases}\qquad \omega_0(s)=\int_0^l K(s,t)\mu(t)\,dt, \tag{12}$$

where

$$K(s,t)=A_t\big[g(\xi(t),\ \ \eta(t);x(s),y(s))\big].$$

Similarly, the single-layer potential (12)

$$\tilde\omega(x,y)=\int_0^l\tilde\mu(t)\,g(\xi(t),\ \ \eta(t);x,y)\,dt, \tag{13}$$

satisfies the jump conditions (11)

$$\begin{cases}A_s[\tilde\omega]_i=\frac{1}{2}\tilde\mu(s)+A_s[\tilde\omega]_0,\\ A_s[\tilde\omega]_e=-\frac{1}{2}\tilde\mu(s)+A_s[\tilde\omega]_0,\end{cases} \tag{14}$$

where

$$A_s[\tilde{\omega}]_0 = \int_0^l K(t,s)\tilde{\mu}(t)\,dt.$$

Energy equalities hold (13), (14):

$$\begin{cases} \displaystyle\iint_{D_1} \left(y^m \tilde{\omega}_x^2 + \tilde{\omega}_y^2\right) dx\,dy = \int_0^l \tilde{\omega}(s)\, A_s[\tilde{\omega}]_i\, ds, \\ \displaystyle\iint_{D_1^c} \left(y^m \tilde{\omega}_x^2 + \tilde{\omega}_y^2\right) dx\,dy = -\int_0^l \tilde{\omega}(s)\, A_s[\tilde{\omega}]_e\, ds. \end{cases} \tag{15}$$

The integral equations for densities μ and $\tilde{\mu}$ are

$$\begin{cases} \displaystyle\mu(s) - \lambda\int_0^l K(s,t)\mu(t)\,dt = \delta(s), \\ \displaystyle\tilde{\mu}(s) - \lambda\int_0^l K(s,t)\tilde{\mu}(t)\,dt = \tilde{\delta}(s), \end{cases} \qquad (\lambda = \pm 2). \tag{16}$$

The kernel $K(s,t)$ has only logarithmic singularity, and Fredholm theory applies. Using the energy identities, one proves that $\lambda = 2$ is not an eigenvalue (15), (16).

Now define the Green function as

$$G(x,y;x_0,y_0) = g(x,y;x_0,y_0) + \vartheta(x,y;x_0,y_0),$$

where ϑ is regular in D_1 and solves

$$\vartheta(\xi(s),\eta(s);x_0,y_0) = -g(\xi(s),\eta(s);x_0,y_0), \qquad \vartheta(x,0;x_0,y_0) = 0.$$

The function ϑ is represented as a double-layer potential:

$$\vartheta(x,y;x_0,y_0) = \int_0^l \mu(t;x_0,y_0)\, A_t[g(\xi(t),\eta(t);x,y)]\, dt,$$

where μ solves (17)

$$\mu(s) - 2\int_0^l K(s,t)\mu(t)\,dt = 2g(\xi(s),\eta(s);x_0,y_0).$$

Since 2 is not an eigenvalue, the solution is

$$\mu(s) = 2g(s) + 4\int_0^l R(s,t;2)\, g(t)\, dt,$$

where $R(s,t;2)$ is the resolvent of K. Substituting this into the formula for ϑ, we obtain

$$\vartheta(x,y;x_0,y_0) = 2\int_0^l g(s;x_0,y_0) A_s[g(s;x,y)]\, ds +$$

$$+4\int_0^l\int_0^l R(s,t;2)\, A_s[g(s;x,y)]\, g(t;x_0,y_0)\, dt\, ds.$$

Finally, the solution of problem (8) is given by

$$z(x_0, y_0) = \int_{-1}^{1} \tau(x) \frac{\partial G(x, 0; x_0, y_0)}{\partial y} \, dx - \int_0^l \big(f(s) - \omega[\eta(s)]\big) A_s[G(s; x_0, y_0)] \, ds.$$

4 Modeling of anomalous diffusion in tumor tissues

The equation

$$\frac{\partial}{\partial x} \left(y^m u_{xx} + u_{yy}\right) = 0,$$

can be interpreted as a mathematical model describing the spatial distribution of the concentration of a biologically active substance - such as a drug, oxygen, or a signaling molecule-in tumor tissue with an anisotropic and heterogeneous structure [17, 18]. The coefficient y^m at the second derivative u_{xx} reflects spatial heterogeneity and the possible degeneration of diffusion [19, 20] along the line $y = 0$.

This may correspond to:

– physiological boundaries within the tissue, for example, the interface between tumor and healthy cells;

– local reduction of permeability or medium density, which may occur due to necrosis, cell compaction, or vascular insufficiency;

– anisotropic properties of the tumor microenvironment, where diffusion along one axis (here, the x-axis) may be significantly restricted depending on the position along the y-axis. Such behavior is characteristic of real tumor structures, where the movement of substances is uneven, and microscopic barriers - such as intercellular spaces and dense tissue regions-create anisotropic diffusion conditions.

This is particularly important for:

– modeling drug delivery into the tumor region, since the equation describes how an agent penetrates into the deeper layers of the tumor;

– analyzing oxygen deprivation (hypoxia) in central tumor zones;

– evaluating the effectiveness of chemo- and radiotherapy, where knowledge of concentration gradients and diffusion conditions helps predict the tumor's response to treatment.

Thus, the presented equation serves as a generalized model of anomalous diffusion in a heterogeneous biological medium and can be useful for constructing quantitative models of tumor growth and therapy.

5 Applied interpretation and numerical illustration

The constructed boundary value problem admits a clear biomedical interpretation. In particular, it can describe the diffusion of a chemical substance or a temperature field in a heterogeneous medium modeling tumor tissue [12, 13]. The function $u(x, y)$ represents the concentration (or intensity) of the distributed quantity, and the parameter $m > 0$ characterizes the degree of anisotropy or inhomogeneity of the medium along the y-axis [14, 16].

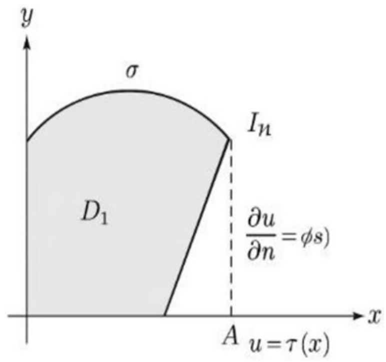

Figure 1 Surface of the analytical solution $u(x, y) = \sin(\pi x)e^{-\pi y}$ in the domain D_1

5.1 Demonstrative example

Consider the simplified boundary value problem

$$\frac{\partial}{\partial x}\big(y^m u_{xx} + u_{yy}\big) = 0, \qquad (x, y) \in D_1,$$

in the unit square

$$D_1 = \{(x, y) \,:\, 0 < x < 1,\ 0 < y < 1\}.$$

for $m = 1$. Boundary conditions are specified as

$$u|_{y=0} = 0, \qquad u|_{y=1} = e^{-x}, \qquad \frac{\partial u}{\partial n}\Big|_{x=0} = y(1 - y).$$

Let us seek the solution in the separable form $u(x, y) = X(x)Y(y)$. Substitution into the equation yields

$$y\,X''(x)Y(y) + X(x)Y''(y) = 0 \quad \Rightarrow \quad \frac{X''(x)}{X(x)} = -\frac{Y''(y)}{y\,Y(y)} = \lambda.$$

Hence,

$$X'' + \lambda X = 0, \qquad Y'' + \lambda y\,Y = 0.$$

For the first eigenvalue $\lambda = \pi^2$, a particular explicit solution is

$$u(x, y) = \sin(\pi x)e^{-\pi y}.$$

This function describes the spatial distribution of a diffusing substance, exhibiting an oscillatory behavior along the x-direction and exponential decay along the y-axis. Such behavior corresponds to attenuation of concentration inside a tumor layer.

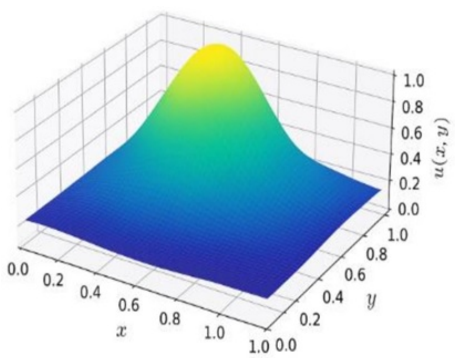

Figure 2 Surface of the analytical solution $u(x, y) = \sin(\pi x)e^{-\pi y}$ in the domain D_1

5.2 Numerical data

To illustrate the solution behavior, Table 1 presents numerical values of

$$u(x, y) = \sin(\pi x)e^{-\pi y},$$

at several representative grid points of the domain D_1.

Table 1 Numerical values of $u(x, y) = \sin(\pi x)e^{-\pi y}$ in the domain D_1.

x	y	$u(x, y)$
0.00	0.00	0.0000
0.25	0.00	0.7071
0.50	0.00	1.0000
0.75	0.00	0.7071
1.00	0.00	0.0000
0.25	0.25	0.3240
0.50	0.25	0.4560
0.75	0.25	0.3240
0.50	0.50	0.2080
0.50	0.75	0.0950

It can be seen that the maximum occurs near the center of the upper boundary, after which the solution decays exponentially along the y-axis, reflecting the diffusion character of the process [21].

5.3 Numerical visualization

Modern computational tools such as **MATLAB**, **COMSOL Multiphysics**, or **Python**-based libraries (e.g., `FEniCS`, `Matplotlib`) can be employed to visualize and analyze the obtained solutions. These environments allow the construction of three-dimensional surfaces of $u(x, y)$, the computation of gradient fields ∇u, as well as the investigation of how the heterogeneity parameter m influences the diffusion dynamics in biological tissues.

6 Conclusion

In this work, a method for solving a boundary value problem for a degenerate elliptic equation was developed using the Green's function and integral equation techniques. The results obtained demonstrate that the proposed approach provides a correct and stable description of the solution under complex geometric constraints and in the presence of degeneracy in the coefficients. An important component of the analysis is the successful application of Fredholm theory to integral equations with kernels of a special form.

The practical significance of the study lies in its applicability to modeling anomalous diffusion processes in heterogeneous and anisotropic media, which is directly related to biomedical problems, including the analysis of substance transport in tumor tissues. Future research may be directed toward extending the theory to broader classes of degenerate equations, as well as performing detailed numerical simulations in geometrically complicated domains.

The analytical and numerical results presented confirm that the developed model adequately captures heterogeneous diffusion behavior and can be further utilized to simulate biomedical phenomena such as drug transport or temperature propagation in tumor tissue.

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УРАВНЕНИЯ СМЕШАННО-СОСТАВНОГО ТИПА В КАЧЕСТВЕ МОДЕЛИ АНОМАЛЬНОЙ ДИФФУЗИИ В ОПУХОЛЕВЫХ ТКАНЯХ

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В данной работе рассматривается краевая задача для вырожденного эллиптического уравнения в плоской области, ограниченной отрезком прямой и аналитической кривой. Основное внимание уделено построению явного аналитического решения с использованием функции Грина, а также методов однослойного и двуслойного потенциалов. Представлено строгое доказательство существования и единственности решения при смешанных граничных условиях. Вырожденность на части границы приводит к существенным аналитическим трудностям, что требует применения продвинутых методов теории эллиптических уравнений с сингулярными коэффициентами. Кроме того, обсуждаются потенциальные приложения модели в биомедицинских задачах, в частности в онкологии. Рассматриваемое уравнение описывает аномальные процессы диффузии в опухолевых тканях с учетом пространственной неоднородности среды. Это делает модель ценным инструментом для анализа распределения лекарственных препаратов, кислорода и других веществ в биологических структурах, характеризующихся выраженной анизотропией и гетерогенностью.

Ключевые слова: функция Грина, потенциал, дуга Жордана, уравнение смешанного типа, условие Липшица, биомедицина, онкология, опухоль, анизотропия.

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