UDC 519.6+51-74:628.395

MATHEMATICAL MODELING OF THE DISPERSION OF HARMFUL SUBSTANCES RELEASED INTO THE ATMOSPHERE IN COMPLEX URBAN ENVIRONMENTS

Ravshanov N., Nasrullaev P., *Boborakhimov B.
*uzbekpy@gmail.com
Digital Technologies and Artificial Intelligence Development Research Institute,
17A, Buz-2, Tashkent, 100125 Uzbekistan.

This study addresses the mathematical modeling of the dispersion process of harmful substances released into the atmosphere in complex urban environments. A mathematical model has been developed to determine the spatial-temporal distribution of pollutant concentrations based on the advection-diffusion equation. The model accounts for the transport of substances by wind flow, turbulent diffusion, gravitational settling, and absorption by surfaces. Mathematical expressions for point, line, and area pollution sources have been presented. Parameters accounting for traffic congestion and diurnal variations have been introduced. A numerical solution algorithm based on the finite difference method has been developed. Computational experiments have revealed the patterns of harmful substance dispersion in urban environments.

Keywords: mathematical model, advection-diffusion equation, atmospheric pollution, turbulent diffusion, gravitational settling, urban environment, numerical methods, traffic congestion.

Citation: Ravshanov N., Nasrullaev P., Boborakhimov B. 2026. Mathematical modeling of the dispersion of harmful substances released into the atmosphere in complex urban environments. *Problems of Computational and Applied Mathematics.* 1(71): 5-15.

DOI: https://doi.org/10.71310/pcam.1_71.2026.01

1 Introduction

The rapid development of global industry and urbanization has led to a year-over-year increase in the amount of harmful substances released into the atmosphere. The dispersion of these substances in the air poses a serious threat to human health and the ecological environment. Particularly in cities, under spatially inhomogeneous conditions, the dispersion of pollutants in air flows between buildings and structures becomes even more complex. Therefore, accurate prediction and monitoring of substance dispersion in such environments is of urgent scientific and practical importance.

Mathematical modeling of the characteristics of the atmospheric surface layer and pollutant transport processes provides the opportunity to study and predict these processes. One of the main tasks in this direction is to develop mathematical models based on numerical methods and implement them in software tools.

Research on mathematical modeling of pollutant transport processes in the atmosphere has a long history, with fundamental works in this field based on fluid dynamics theory. Regarding the application of turbulence models to industrial problems, the $k-\varepsilon$ model developed by Launder and Spalding [1] has become widely used. This model enables the determination of turbulent viscosity by accounting for turbulent kinetic energy and its dissipation rate.

The $k-\omega$ model developed by Wilcox [2] has been recognized as an effective tool for modeling boundary layer flows. In this model, turbulent viscosity is determined using turbulent kinetic energy k and specific dissipation rate ω, which enables high-accuracy calculation of near-wall flows. In the direction of improving turbulence models, Menter [3] developed the SST (Shear Stress Transport) model, which combines the advantages of both $k-\omega$ and $k-\varepsilon$ models.

Regarding numerical methods and discretization schemes, the fundamental work by Ferziger and Peric [4] provides detailed coverage of applying finite difference and finite volume methods to fluid flow problems. Patankar [5] developed effective numerical schemes for calculating advective-diffusive processes, with his "upwind"differencing scheme ensuring stability and accuracy. Versteeg and Malalasekera [6] systematically presented the theoretical foundations and practical applications of the finite volume method.

The fundamental work on atmospheric chemistry and physics by Seinfeld and Pandis [7] provides detailed coverage of chemical reactions in the atmosphere, aerosol dynamics, and the patterns of pollutant dispersion. Regarding atmospheric pollution prediction, the works of Berlyand [8] are of particular importance, and his developed methods have been adopted as standards for calculating the dispersion of harmful substances around industrial enterprises.

Significant achievements have been made in recent years in the field of modeling air pollution in urban environments. Zhong et al. [9] presented a comprehensive review on simulating microscale air flows and pollutant dispersion in urban environments based on CFD models. Their analysis identified the effects of factors such as building geometry, atmospheric boundary layer stability, street canyon aspect ratios, and tree configurations on microscale physical processes.

Dimitrova et al. [10] conducted a systematic review on modeling air pollution in urban environments using CFD, analyzing more than 90 scientific works from the last decade. They categorized modeling features into 7 thematic groups, systematizing more than 190 parameters. This research identified the main trends in CFD model applications in urban environments.

Regarding modeling the dispersion of traffic emissions in urban environments, Ioannidis et al. [11] studied the dispersion of PM, CO, and NOx gases from traffic activity in the city of Augsburg, Germany, using a CFD model. Their model, combining the RANS approach with the advection-diffusion equation, provided highly accurate results.

Saleh et al. [12] developed a modified $k-\omega$ turbulence model for neutral atmospheric boundary layer flows. They eliminated inconsistency with inlet conditions by introducing additional source terms to the standard model. This model enables high-accuracy prediction of flow and pollution dispersion in complex urban geometries.

Lin et al. [13] developed a method for modeling pollutant dispersion at the street scale by combining chemical reactions, aerosol dynamics, and CFD. By integrating the SSH-aerosol module with OpenFOAM and Code_Saturne software, they achieved higher accuracy in calculating NO2, PM10, and black carbon concentrations compared to conventional CFD simulations.

Garcia et al. [14] developed a CFD model for studying PM10 dispersion in large-scale open spaces. Their model enabled simulation of pollutant dispersion under various wind conditions over a 12x18 km^2 area. The research identified the influence of terrain features on dispersion in open areas.

Regarding modeling heavy gas dispersion in urban environments, Marek et al. [15] developed a modified $k-\varepsilon$ turbulence model on the OpenFOAM platform. This model correctly predicts atmospheric boundary layer flow while also accounting for obstacles in the area.

Regarding the study of building configuration and vegetation effects in urban environments, Chen et al. [16] investigated air flows, CO dispersion, and population exposure in various 3D urban morphologies through CFD simulations. They applied the Building Intake Fraction parameter for exposure assessment.

Mirzaei [17] analyzed the problems and prospects of CFD modeling of urban microclimate and climate, identifying tasks that need to be solved in the coming decade. His analysis showed the need for development in directions such as mesh optimization, integration with mesoscale models, and simulation of various weather conditions.

In the direction of combining artificial intelligence and machine learning methods with turbulence modeling, Duraisamy et al. [18] proposed data-driven approaches for improving the accuracy of RANS models. This direction opens new possibilities in the field of turbulent flow modeling.

Wai et al. [19] combined artificial neural networks and CFD models to predict PM_{10} dispersion from traffic emissions in dense high-rise urban environments. This hybrid approach ensured high accuracy and computational efficiency.

Among the works of local scientists, Ravshanov et al. [20] conducted research on modeling and predicting the dispersion of harmful substances in industrial area atmospheres. Sharipov et al. [21] developed numerical modeling methods for short-term air quality forecasting in industrial regions.

The research conducted by Boborakhimov [22] on numerical modeling of turbulent transport of pollutants in spatially inhomogeneous atmospheric environments is of particular importance. In his work, a method for determining wind fields in complex geometric regions based on Navier-Stokes equations and the $k-\omega$ turbulence model was developed. This approach enables high-accuracy calculation of flow characteristics around buildings in urban environments and has been adopted as the basis for determining the wind field in this study.

Modern development trends in turbulence models have been summarized by Durbin [23]. His analysis identified directions for combining RANS and LES models, as well as developing hybrid approaches. The research by Britter and Hanna [24] on flow and dispersion characteristics in urban environments is of great importance, as they studied recirculation zones in urban canyons and pollutant dispersion patterns both experimentally and theoretically. The theoretical foundations of diffusion processes in the atmosphere have been fundamentally covered by Pasquill and Smith [25].

The above literature review shows that modeling the dispersion of harmful substances in the atmosphere is a complex multifactorial process that is carried out by combining knowledge of turbulent flow theory, numerical methods, and atmospheric physics. In urban environments, this problem becomes even more complex because buildings and structures significantly alter the flow structure. Therefore, this study aims to develop a mathematical model that accounts for complex urban geometric conditions.

2 Problem Formulation

Mathematical modeling of the turbulent dispersion process of harmful substances in spatially inhomogeneous atmospheric environments requires accounting for the characteristics of air flows in complex urban geometric conditions. In urban environments, due to the presence of buildings, structures, and other obstacles, the wind field has a complex structure, which significantly changes the dispersion characteristics of harmful substances.

In this study, the Navier–Stokes equations and the k–ω turbulence model are used to determine the wind field components (u, v, w). This approach is presented in detail in [22], and in this work, attention is focused on studying the dispersion patterns of harmful substance concentrations using the obtained velocity field.

The generalized advection–diffusion equation describing the dispersion of a single type of harmful substance in the atmosphere has the following form [7, 8]:

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + (w - w_g)\frac{\partial C}{\partial z} =$$
$$= \frac{\partial}{\partial x}\left(K_x\frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y\frac{\partial C}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z\frac{\partial C}{\partial z}\right) + S - \lambda C - \sigma C.$$

In this equation, $C = C(x, y, z, t)$ is the concentration of harmful substance in the atmosphere (kg/m^3); t is time (s); x, y, z are Cartesian coordinates, where the x – axis is directed east, the y-axis north, and the z-axis vertically upward (m); u, v, w are wind velocity components along the x, y, and z axes, respectively (m/s); w_g is the gravitational settling velocity of particles (m/s); K_x, K_y, K_z are turbulent diffusion coefficients (m^2/s); S is the pollution source intensity (kg/(m^{3}s)); λ is the chemical decay coefficient of the substance (1/s); σ is the absorption coefficient of the substance by surfaces (1/s).

The first term on the left side of the equation represents local change over time, while the next three terms represent the advection process, i.e., the transport of substance along with the wind flow. On the right side, the diffusion terms represent the dispersion of substance due to turbulent mixing, the source term represents pollutants released into the atmosphere, and the last two terms represent chemical decay of the substance and absorption by surfaces [8].

The advection process describes the transport of substance along with the wind flow and has several specific characteristics in urban environments [24]. In narrow streets between tall buildings, the "canyon effect"is observed, where wind direction and velocity change significantly, creating recirculation zones. The following parameter is used to assess the effect of wind direction on pollution dispersion zones. The following parameter is used to assess the effect of wind direction on pollution dispersion:

$$\xi = \frac{U \cdot |\cos\theta|}{u_g},$$

U is the wind velocity in open areas (m/s); θ is the angle of wind relative to the street axis (rad); u_g is the average wind velocity inside the street canyon (m/s). When $\xi > 1$, the city is well ventilated and pollution disperses quickly; when $\xi < 1$, pollution accumulates in street canyons.

The diffusion process describes the dispersion of substance due to turbulent mixing [25]. Turbulent diffusion coefficients are determined through turbulent viscosity:

$$K_x = K_y = \frac{\nu_t}{Sc_t},$$

$$K_z = \frac{\nu_t}{Sc_t} \cdot \psi(z).$$

Here, $\nu_t - k/\omega$ is turbulent viscosity, calculated based on the $k-\omega$ model [2, 22] (m^2/s); $Sc_t = 0.7 - 1.0.$ is the turbulent Schmidt number (dimensionless); $\psi(z)$ is the vertical stratification function accounting for atmospheric stability (dimensionless). Depending on atmospheric conditions, the stratification function takes the following values: under unstable conditions $\psi(z) = 1 + 0.5\,(z/L_M)$, under neutral conditions $\psi(z) = 1$, under stable conditions $\psi(z) = 1 - 0.5\,(z/L_M)$, where L_M is the Monin-Obukhov length (m).

The gravitational settling process describes the settling of heavy particles toward the ground. The settling velocity is calculated based on Stokes' formula [7]:

$$w_g = \frac{\rho_p \cdot g \cdot d_p^2}{18 \cdot \mu} \cdot C_c.$$

Here, ρ_p is particle density (kg/m^3); $g = 9.81$ is gravitational acceleration (m/s^2); d_p is particle diameter (m); μ is dynamic viscosity of air (Pa·s); C_c is the Cunningham correction factor (dimensionless). For fine particles, the Cunningham correction factor is determined as follows:

$$C_c = 1 + \frac{2\lambda}{d_p}\left(1.257 + 0.4 \exp\left(-\frac{1.1 d_p}{2\lambda}\right)\right).$$

Here, $\lambda \approx 0.065$ μm is the mean free path of air. For different fractions, settling velocities take the following values: for PM2.5 fraction $w_g = 0.0001 - 0.001$ m/s, for PM10 fraction $w_g = 0.001 - 0.01$ m/s, for coarse dust particles $w_g = 0.01 - 0.1$ m/s.

The absorption process describes the adsorption of harmful substances by various surfaces [16]. The total absorption coefficient consists of the following components:

$$\sigma = \sigma_o + \sigma_b + \sigma_y.$$

Here, σ_o is absorption by vegetation, σ_b is absorption by building surfaces, σ_y is absorption by ground surface coefficients (1/s). The vegetation absorption coefficient is determined as follows:

$$\sigma_o = \frac{v_o \cdot LAI}{H_o}.$$

Here, v_o is deposition velocity to vegetation surface (m/s); LAI is leaf area index (m^2/m^2); H_o is vegetation canopy height (m). The building surface absorption coefficient:

$$\sigma_b = \frac{v_d \cdot A_b}{V_t}.$$

Here, v_d is deposition velocity to building surface (m/s); A_b is building surface area (m^2); V_i is influence zone volume (m^3).

area (m^2); V_i is influence zone volume (m^3).

Pollution sources are represented in three forms in the model [8, 11]. Point sources describe gases released from industrial enterprise stacks:

$$S_n = \frac{M}{V_k} \cdot \delta(x - x_m) \cdot \delta(y - y_m) \cdot \delta(z - z_s).$$

Here, M is substance emission rate from the source (kg/s); (x_m, y_m, z_s) are source coordinates (m); V_k is computational cell volume (m^3); δ is the Dirac delta function. For stacks emitting hot gases, the effective height is determined as follows:

$$z_s = h + \Delta h,$$

$$\Delta h = \frac{1.6 \cdot F^{1/3} \cdot X^{2/3}}{U}.$$

Here, h is the physical height of the stack (m); Δh is the rise height of gas due to buoyancy (m); U is wind velocity (m/s); X is the horizontal distance at which maximum rise is achieved (m). The buoyancy parameter is calculated as follows:

$$F = g \cdot V_g \cdot \frac{d^2}{4} \cdot \frac{T_g - T_a}{T_g}.$$

Here, V_g is gas exit velocity from the stack (m/s); d is stack diameter (m); T_g is temperature of emitted gas (K); T_a is atmospheric temperature (K).

Line sources describe vehicles located along roads [11]:

$$S_l = \frac{E \cdot N}{V \cdot \Delta y \cdot \Delta z} \cdot \eta\left(t\right) \cdot \eta_t \cdot \chi\left(x, y\right).$$

Here, E is average emission per vehicle (kg/(veh·s)); N is number of vehicles on the road (veh); V is average traffic flow velocity (m/s); $\eta(t)$ is time-dependent coefficient (dimensionless); η_t is congestion coefficient (dimensionless); $\chi(x, y)$ is characteristic function defining the road area.

Under traffic congestion conditions, vehicle speeds decrease sharply and emissions increase:

$$\eta_t = 1 + \alpha_t \cdot \left(\frac{\rho_t}{\rho_m}\right)^{\beta}.$$

Here, ρ_t is current traffic density (veh/km); ρ_m is maximum road capacity (veh/km); $\alpha_t = 2.5 - 4.0$ is congestion intensity coefficient (dimensionless); $\beta = 1.5 - 2.0$ is power exponent (dimensionless).

Wind direction determines the penetration distance of traffic emissions into the city:

$$L_p = \frac{C_0 \cdot U \cdot |\cos\theta|}{\sigma \cdot u_*}.$$

Here, L_p is penetration distance (m); C_0 is concentration above the road (kg/m^3); u_* is friction velocity (m/s).

Area sources describe construction sites and open industrial areas:

$$S_m = \frac{E_m \cdot A_m}{V_k} \cdot \chi\left(x, y, z\right).$$

Here, E_m is emission intensity per unit area (kg/(m^2·s)); A_m is source area (m^2).

Initial and boundary conditions are required to find the solution of the problem. Initial condition:

$$C(x, y, z, 0) = C_f(x, y, z), \quad (x, y, z) \in \Omega_h.$$

At the inlet boundary ($x = 0$):

$$C = C_f.$$

At the outlet boundary ($x = L_x$):

$$\frac{\partial C}{\partial x} = 0.$$

At lateral boundaries ($y = 0$ and $y = L_y$):

$$\frac{\partial C}{\partial y} = 0.$$

At the upper boundary ($z = L_z$):

$$\frac{\partial C}{\partial z} = 0.$$

At the ground surface boundary ($z = 0$):

$$-K_z \frac{\partial C}{\partial z} = v_g \cdot C.$$

$$-K_z \frac{\partial C}{\partial z} = v_d \cdot C.$$

At building surface boundaries:

$$-K_n \frac{\partial C}{\partial n} = v_d \cdot C.$$

Here, v_d is deposition velocity to building surface, with values of $v_d = 1.2 \cdot v_g$ for windward surfaces, $v_d = 1.5 \cdot v_g$ for leeward surfaces, $v_d = v_g$ for lateral surfaces, and $v_d = 2.0 \cdot v_g$ for roof surfaces.

3 Solution of the problem

The finite difference method is applied for numerical investigation of the above equations [4, 5]. Unknown functions in each equation are considered on the following computational grid:

$$\Omega_{xyzt} = \left\{ (x_i, y_j, z_k, t_n)\,; i = \overline{1, N_x}; j = \overline{1, N_y}; k = \overline{1, N_z}; n = \overline{0, N_t} \right\}.$$

Here, $x_i = i\Delta x_i, y_j = j\Delta y_j, z_k = k\Delta z_k, t_n = n\Delta t$.

The time step is determined considering stability conditions [6]:

$$\Delta t_f = \min\left(\frac{\Delta x}{\max(u_f)}, \frac{\Delta y}{\max(v_f)}, \frac{\Delta z}{\max(w_f)}, \frac{\Delta x^2}{2K_{max}}, \frac{\Delta y^2}{2K_{max}}, \frac{\Delta z^2}{2K_{max}} \right).$$

Here, $K_{max} = \max(K_x, K_y, K_z)$ is the maximum diffusion coefficient.

Initial conditions are given as:

$$C(x, y, z, 0) = C_0.$$

For numerical solution of the advection-diffusion equation, the temporary concentration field is calculated as follows:

$$C^{n+1}_{i,j,k} = C^n_{i,j,k} - \Delta t\,(A_x + A_y + A_z) +$$
$$+\Delta t\,(D_x + D_y + D_z) + \Delta t \cdot S^n_{i,j,k} - \Delta t\,(\lambda + \sigma)\, C^n_{i,j,k}.$$

Here, advection terms are discretized using the "upwind"scheme [5]:

$$A_x = \begin{cases} u^{n+1}_{i,j,k}\dfrac{C^n_{i,j,k} - C^n_{i-1,j,k}}{\Delta x}, \ u^{n+1}_{i,j,k} > 0, \\ u^{n+1}_{i,j,k}\dfrac{C^n_{i+1,j,k} - C^n_{i,j,k}}{\Delta x}, \ u^{n+1}_{i,j,k} < 0, \end{cases}$$

$$A_y = \begin{cases} v^{n+1}_{i,j,k}\dfrac{C^n_{i,j,k} - C^n_{i,j-1,k}}{\Delta y}, \ v^{n+1}_{i,j,k} > 0, \\ v^{n+1}_{i,j,k}\dfrac{C^n_{i,j+1,k} - C^n_{i,j,k}}{\Delta y}, \ v^{n+1}_{i,j,k} < 0, \end{cases}$$

$$A_z = \begin{cases} \left(w^{n+1}_{i,j,k} - w_g\right)\dfrac{C^n_{i,j,k} - C^n_{i,j,k-1}}{\Delta z}, \ w^{n+1}_{i,j,k} - w_g > 0, \\ \left(w^{n+1}_{i,j,k} - w_g\right)\dfrac{C^n_{i,j,k+1} - C^n_{i,j,k}}{\Delta z}, \ w^{n+1}_{i,j,k} - w_g < 0. \end{cases}$$

Diffusion terms are discretized using central differences [4]:

$$D_x = \frac{1}{\Delta x}\left[K_{x,i+1/2,j,k}\frac{C^n_{i+1,j,k} - C^n_{i,j,k}}{\Delta x} - K_{x,i-1/2,j,k}\frac{C^n_{i,j,k} - C^n_{i-1,j,k}}{\Delta x}\right],$$

$$D_y = \frac{1}{\Delta y}\left[K_{y,i,j+1/2,k}\frac{C^n_{i,j+1,k} - C^n_{i,j,k}}{\Delta y} - K_{y,i,j-1/2,k}\frac{C^n_{i,j,k} - C^n_{i,j-1,k}}{\Delta y}\right],$$

$$D_z = \frac{1}{\Delta z}\left[K_{z,i,j,k+1/2}\frac{C^n_{i,j,k+1} - C^n_{i,j,k}}{\Delta z} - K_{z,i,j,k-1/2}\frac{C^n_{i,j,k} - C^n_{i,j,k-1}}{\Delta z}\right].$$

Values at boundary points are determined as follows:

$$C^{n+1}_{0,j,k} = C_f, \ C^{n+1}_{N_x,j,k} = C^{n+1}_{N_x-1,j,k},$$

$$C^{n+1}_{i,0,k} = C^{n+1}_{i,1,k}, \ C^{n+1}_{i,N_y,k} = C^{n+1}_{i,N_y-1,k},$$

$$C^{n+1}_{i,j,N_z} = C^{n+1}_{i,j,N_z-1}.$$

For the ground surface:

$$C^{n+1}_{i,j,0} = \frac{K_z \cdot C^{n+1}_{i,j,1}}{K_z + v_g \cdot \Delta z}.$$

For convergence checking, the residual value is calculated:

$$R_C = \max_{i,j,k}\left|\frac{C^{n+1}_{i,j,k} - C^n_{i,j,k}}{C^{n+1}_{i,j,k} + \varepsilon}\right|.$$

Here, ε is a small number for protection against division by zero. If $R_C < R_{tol}$, the iteration process is terminated.

4 Results and discussion

Computational experiments were conducted to verify the suitability of the developed mathematical apparatus. For this purpose, the considered mathematical model and numerical algorithm were implemented as software written in the Python programming language.

The following conditions and constraints were adopted for modeling the dispersion process of pollutants in the atmospheric surface layer, taking into account urban planningelements. The computational domain was taken with dimensions of $900 \times 300 \times 100$ m, with buildings of various heights placed within it.

Carbon dioxide CO_2 gas emitted from vehicles moving along the road was taken as the pollutant. Wind velocity was set at $U = 3$ m/s, with direction from west to east, i.e., $\varphi = 0°$.

The following patterns were identified as a result of computational experiments. First, recirculation zones formed in narrow streets between buildings, where concentrations were observed to be 2-3 times higher than in surrounding open areas. Second, the shape and direction of the pollution plume changed significantly depending on wind direction. Third, concentrations reached maximum values during traffic congestion hours.

The cleaning efficiency of green areas was analyzed. A 15-25% decrease in concentration was found in areas covered with trees. It was confirmed that coniferous trees have higher absorption capacity compared to deciduous trees.

Calculations were performed under various atmospheric stability conditions. Under stable conditions (inversion), vertical mixing is weak, and pollution accumulates in layers close to the ground surface. Under unstable conditions, vertical mixing is strong, and pollution disperses more quickly.

5 Conclusion

As a result of the research, a mathematical model describing the dispersion of harmful substances released into the atmosphere in complex urban geometric environments was developed. The model jointly accounts for advection, diffusion, gravitational settling, and absorption by surfaces processes. Mathematical expressions for point, line, and area pollution sources were presented. Parameters accounting for traffic congestion and diurnal variations were introduced, which expands the practical applicability of the model. A numerical solution algorithm based on the finite difference method was developed and implemented in the Python programming language. The conducted computational experiments confirmed the adequacy of the model and its practical application possibilities. The developed model can be applied in urban planning, environmental monitoring, and air quality management problems.

References

[1] Launder B.E., Spalding D.B. The numerical computation of turbulent flows // *Computer Methods in Applied Mechanics and Engineering.* – 1974. – Vol. 3. – P. 269-289.

[2] Wilcox D.C. *Turbulence Modeling for CFD.* – 3rd ed. – La Cañada, CA: DCW Industries, 2006. – 522 p.

[3] Menter F.R. Two-equation eddy-viscosity turbulence models for engineering applications // *AIAA Journal.* – 1994. – Vol. 32. – P. 1598-1605.

[4] Ferziger J.H., Perić M. *Computational Methods for Fluid Dynamics.* – 3rd ed. – Berlin: Springer, 2002. – 423 p.

[5] Patankar S.V. *Numerical Heat Transfer and Fluid Flow.* – Washington: Hemisphere Publishing, 1980. – 197 p.

[6] Versteeg H.K., Malalasekera W. *An Introduction to Computational Fluid Dynamics: The Finite Volume Method.* – 2nd ed. – Harlow: Pearson Education, 2007. – 517 p.

[7] Seinfeld J.H., Pandis S.N. *Atmospheric Chemistry and Physics: From Air Pollution to Climate Change.* – 3rd ed. – Hoboken, NJ: John Wiley & Sons, 2016. – 1152 p.

[8] Berlyand M.E. *Prediction and Regulation of Atmospheric Pollution.* – Leningrad: Gidrometeoizdat, 1985. – 272 p. (in Russian)

[9] Zhong J., Cai X., Bloss W.J. Simulating microscale urban airflow and pollutant distributions based on computational fluid dynamics model: A review // *Atmosphere.* – 2023. – Vol. 14. – Art. no. 1727.

[10] Dimitrova R., Velizarova M., Tsekov M. Air pollution dispersion modelling in urban environment using CFD: A systematic review // *Atmosphere.* – 2022. – Vol. 13. – Art. no. 1640.

[11] Ioannidis G., Li C., Tremper P., Riedel T., Ntziachristos L. Application of CFD modelling for pollutant dispersion at an urban traffic hotspot // *Atmosphere.* – 2024. – Vol. 15. – Art. no. 113.

[12] Saleh A., Lakkis I., Moukalled F. A modified k-ω turbulence model for improved predictions of neutral atmospheric boundary layer flows // *Building and Environment.* – 2022. – Vol. 223. – Art. no. 109495.

[13] Lin C., Wang Y., Ooka R., Flageul C., Kim Y., Kikumoto H., Wang Z., Sartelet K. Modeling of street-scale pollutant dispersion by coupled simulation of chemical reaction, aerosol dynamics, and CFD // *Atmospheric Chemistry and Physics.* – 2023. – Vol. 23. – P. 1421-1436.

[14] Garcia J.A., Coz-Diaz J.J., Alvarez-Rabanal F.P., Garcia-Nieto P.J. CFD model to study PM10 dispersion in large-scale open spaces // *Scientific Reports.* – 2023. – Vol. 13. – Art. no. 5966.

[15] Marek M., Jaroszynski P. A modified k-ε turbulence model for heavy gas dispersion in built-up environment // *Atmosphere.* – 2023. – Vol. 14. – Art. no. 161.

[16] Chen G., Rong L., Zhang G. Numerical investigations of urban pollutant dispersion and building intake fraction with various 3D building configurations and tree plantings // *International Journal of Environmental Research and Public Health.* – 2022. – Vol. 19. – Art. no. 3524.

[17] Mirzaei P.A. CFD modeling of micro and urban climates: Problems to be solved in the new decade // *Sustainable Cities and Society.* – 2021. – Vol. 69. – Art. no. 102839.

[18] Duraisamy K., Iaccarino G., Xiao H. Turbulence modeling in the age of data // *Annual Review of Fluid Mechanics.* – 2019. – Vol. 51. – P. 357-377.

[19] Wai K.M., Yu P.K.N., Wong C.K.C. Machine learning and CFD for PM10 dispersion modeling in dense urban environment // *Building and Environment.* – 2023. – Vol. 228. – Art. no. 109831.

[20] Ravshanov N., Sharipov D.K., Muradov F. Modeling and prediction of the dispersion of harmful substances released into the atmosphere of industrial areas // *Problems of Computational and Applied Mathematics.* – 2017. – № 6(12). – P. 41-52. (in Uzbek)

[21] Sharipov D.K., Muradov F., Ravshanov N. Numerical modeling method for short-term air quality forecast in industrial regions // *Journal of Physics: Conference Series.* – 2019. – Vol. 1260. – Art. no. 102012.

[22] Boborakhimov B.I. Numerical modeling of turbulent transport of impurities in a spatially inhomogeneous atmospheric environment // *Problems of Computational and Applied Mathematics.* – 2025. – № 1(63). – P. 77-98.

[23] Durbin P.A. Some recent developments in turbulence closure modeling // *Annual Review of Fluid Mechanics.* – 2018. – Vol. 50. – P. 77-103.

[24] Britter R.E., Hanna S.R. Flow and dispersion in urban areas // *Annual Review of Fluid Mechanics.* – 2003. – Vol. 35. – P. 469-496.

[25] Pasquill F., Smith F.B. *Atmospheric Diffusion.* – 3rd ed. – Chichester: Ellis Horwood, 1983. – 437 p.

УДК 519.6+51-74:628.395

МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ РАССЕИВАНИЯ ВРЕДНЫХ ВЕЩЕСТВ, ВЫБРАСЫВАЕМЫХ В АТМОСФЕРУ В УСЛОВИЯХ СЛОЖНОЙ ГОРОДСКОЙ СРЕДЫ

Равшанов Н., Насруллаев П., *Боборахимов Б.

*uzbekpy@gmail.com

Научно-исследовательский институт развития цифровых технологий и искусственного интеллекта,

100125, Узбекистан, г. Ташкент, Мирзо-Улугбекский р-он, м-в Буз-2, д. 17А.

В данном исследовании рассматривается математическое моделирование процесса дисперсии вредных веществ, выбрасываемых в атмосферу в сложных городских условиях. Разработана математическая модель для определения пространственно-временного распределения концентраций загрязняющих веществ на основе уравнения адвекции-диффузии. Модель учитывает перенос веществ потоком ветра, турбулентную диффузию, гравитационное осаждение и абсорбцию поверхностями. Представлены математические выражения для источников загрязнения точек, линий и площадей. Введены параметры, учитывающие загруженность дорог и суточные колебания. Разработан алгоритм численного решения, основанный на методе конечных разностей. Вычислительные эксперименты выявили закономерности дисперсии вредных веществ в городской среде.

Ключевые слова: математическая модель, адвективно-диффузионное уравнение, загрязнение атмосферы, турбулентная диффузия, гравитационное осаждение, городская среда, численные методы, пробки.

Цитирование: *Равшанов Н., Насруллаев П., Боборахимов Б.* Математическое моделирование рассеивания вредных веществ, выбрасываемых в атмосферу в условиях сложной городской среды // Проблемы вычислительной и прикладной математики. – 2026. – № 1(71). – С. 5-15.

DOI: https://doi.org/10.71310/pcam.1_71.2026.01

№ 1(71) 2026

ISSN 2181-8460

HISOBLASH VA AMALIY MATEMATIKA MUAMMOLARI

ПРОБЛЕМЫ ВЫЧИСЛИТЕЛЬНОЙ И ПРИКЛАДНОЙ МАТЕМАТИКИ

PROBLEMS OF COMPUTATIONAL AND APPLIED MATHEMATICS

RAQAMLI TEXNOLOGIYALAR VA
SUN'IY INTELLEKTNI RIVOJLANTIRISH
ILMIY-TADQIQOT INSTITUTI

ПРОБЛЕМЫ ВЫЧИСЛИТЕЛЬНОЙ И ПРИКЛАДНОЙ МАТЕМАТИКИ

№ 1(71) 2026

Журнал основан в 2015 году.
Издается 6 раз в год.

Учредитель:
Научно-исследовательский институт развития цифровых технологий и искусственного интеллекта.

Главный редактор:
Равшанов Н.

Заместители главного редактора:
Арипов М.М., Шадиметов Х.М., Ахмедов Д.Д.

Ответственный секретарь:
Убайдуллаев М.Ш.

Редакционный совет:
Азамов А.А., Алоев Р.Д., Амиргалиев Е.Н. (Казахстан), Арушанов М.Л., Бурнашев В.Ф., Джумаёзов У.З., Загребина С.А. (Россия), Задорин А.И. (Россия), Игнатьев Н.А., Ильин В.П. (Россия), Иманкулов Т.С. (Казахстан), Исмагилов И.И. (Россия), Кабанихин С.И. (Россия), Карачик В.В. (Россия), Курбонов Н.М., Маматов Н.С., Мирзаев Н.М., Мухамадиев А.Ш., Назирова Э.Ш., Нормуродов Ч.Б., Нуралиев Ф.М., Опанасенко В.Н. (Украина), Расулмухамедов М.М., Садуллаева Ш.А., Старовойтов В.В. (Беларусь), Хаётов А.Р., Халджигитов А., Хамдамов Р.Х., Хужаев И.К., Хужаеров Б.Х., Эшмаматова Д.Б., Дустмуродова Ш.Ж., Чье Ен Ун (Россия), Шабозов М.Ш. (Таджикистан), Dimov I. (Болгария), Li Y. (США), Mascagni M. (США), Min A. (Германия), Singh M. (Южная Корея).

Журнал зарегистрирован в Агентстве информации и массовых коммуникаций при Администрации Президента Республики Узбекистан.
Свидетельство №0856 от 5 августа 2015 года.

ISSN 2181-8460, eISSN 2181-046X

При перепечатке материалов ссылка на журнал обязательна.
За точность фактов и достоверность информации ответственность несут авторы.

Адрес редакции:
100125, г. Ташкент, м-в. Буз-2, 17А.
Тел.: +(998) 71 263-41-98.
Э-почта: journals@airi.uz.
Веб-сайт: https://journals.airi.uz.

Дизайн и вёрстка:
Шарипов Х.Д.

Отпечатано в типографии НИИ РЦТИИ.
Подписано в печать 25.02.2026 г.
Формат 60x84 1/8. Заказ №1. Тираж 100 экз.

PROBLEMS OF COMPUTATIONAL AND APPLIED MATHEMATICS

No. 1(71) 2026

The journal was established in 2015.
6 issues are published per year.

Founder:
Digital Technologies and Artificial Intelligence Development Research Institute.

Editor-in-Chief:
Ravshanov N.

Deputy Editors:
Aripov M.M., Shadimetov Kh.M., Akhmedov D.D.

Executive Secretary:
Ubaydullaev M.Sh.

Editorial Council:
Azamov A.A., Aloev R.D., Amirgaliev E.N. (Kazakhstan), Arushanov M.L., Burnashev V.F., Djumayozov U.Z., Zagrebina S.A. (Russia), Zadorin A.I. (Russia), Ignatiev N.A., Ilyin V.P. (Russia), Imankulov T.S. (Kazakhstan), Ismagilov I.I. (Russia), Kabanikhin S.I. (Russia), Karachik V.V. (Russia), Kurbonov N.M., Mamatov N.S., Mirzaev N.M.,Mukhamadiev A.Sh., Nazirova E.Sh., Normurodov Ch.B., Nuraliev F.M., Opanasenko V.N. (Ukraine), Sadullaeva Sh.A., Starovoitov V.V. (Belarus), Khayotov A.R., Khaldjigitov A., Khamdamov R.Kh., Khujaev I.K., Khujayorov B.Kh., Eshmamatova D.B., Dustmurodova Sh.J., Chye En Un (Russia), Shabozov M.Sh. (Tajikistan), Dimov I. (Bulgaria), Li Y. (USA), Mascagni M. (USA), Min A. (Germany), Singh M. (South Korea).

The journal is registered by Agency of Information and Mass Communications under the Administration of the President of the Republic of Uzbekistan.
Certificate of Registration No. 0856 of 5 August 2015.

ISSN 2181-8460, eISSN 2181-046X

At a reprint of materials the reference to the journal is obligatory.
Authors are responsible for the accuracy of the facts and reliability of the information.

Address:
100125, Tashkent, Buz-2, 17A.
Tel.: +(998) 71 263-41-98.
E-mail: journals@airi.uz.
Web-site: https://journals.airi.uz.

Layout design:
Sharipov Kh.D.

DTAIRI printing office.
Signed for print 25.02.2026
Format 60x84 1/8. Order No. 1. Print run of 100 copies.

Содержание

Contents