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DESIGN AND OPERATING PRINCIPLE OF A COGENERATIVE FRACTAL SOLAR COLLECTOR

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For the first time, a fractal solar collector operating in the cogeneration mode is proposed. In this context, we are talking about a modern innovative system that uses fractal structures to optimize the processes of collecting, distributing and converting solar energy simultaneously into thermal and electrical energy. The detailed design and results of the conducted experiment of this system are given. The stages of the experiment are shown in order to gain a deeper understanding of the principles of operation of a collector operating in the cogeneration mode. The high efficiency and promising application of this system in the field of renewable energy have been experimentally proven. In addition, the results obtained demonstrate the possibility of using the proposed fractal collector as an alternative energy source, which opens up new ways to develop energy-efficient and environmentally friendly technologies of the future.

Keywords: solar collector, fractal, cogeneration, transformations, solar energy, energy generation, cogeneration mode.

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1 Introduction

In recent years, research on new-generation solar collectors has intensified, allowing for more efficient use of solar energy. Research is ongoing to improve widely used flat solar collectors [1, 11].

However, studies of enrichment systems to increase efficiency are also becoming increasingly important, and parabolic grooved reservoirs stand out in this area [2–4]. In addition to developing new approaches to the design of these collectors [5, 10], various methods for improving the performance of parabolic grooved collectors are also being investigated. Such methods include improving the design of the absorber surface [6, 7], and the study of various heat carriers [8, 9].

There is no cogeneration mode in these considered reservoirs. In its physical essence, when a fractal collector operates in cogeneration mode, its efficiency increases significantly [12, 13].

The aim of the work is to develop a fractal solar collector operating in the cogeneration mode.

2 The solution method

There are great opportunities for using solar panels, not only for private clients. These installations can be used in the regional energy generation system (AEG). Scaling up the use of solar installations will not only significantly save energy resources, but also alleviate the environmental situation. Solar water heating systems with absorbers fractally located in a parabolic concentrate, i.e., a Cogenerative fractal solar collector (CFSC), are structurally simple [14].

Thus, the desired result from CFSC during operation is achieved by increasing the efficiency of using sunlight in the space of a parabolic concentrator in which fractally arranged toroidal absorbers, as well as optimizing the location of fractal toroidal absorbers to obtain warm water of different temperatures. In this case, the aperture area of the CFSC will be the area of the parabolic concentrator (Fig. 1).

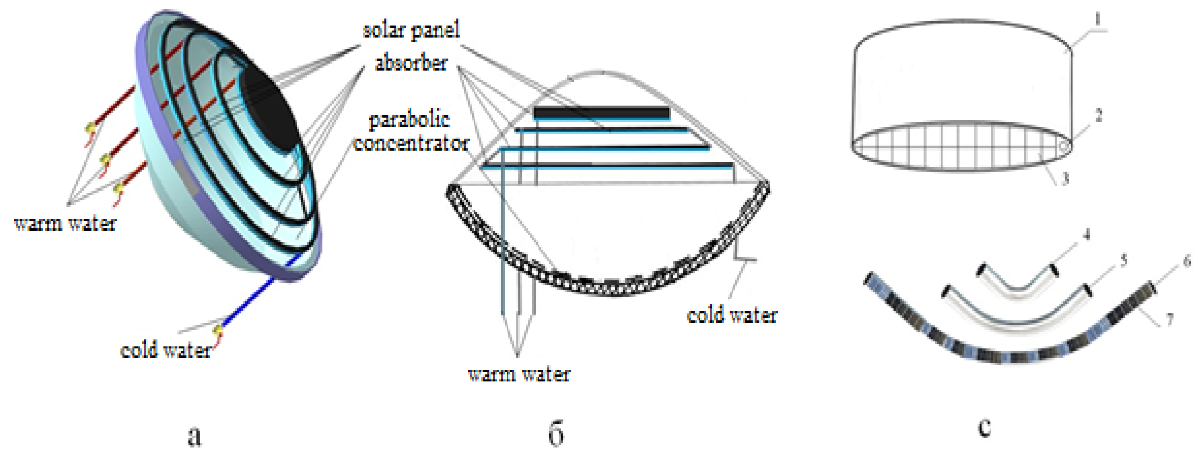

Figure 1 The design of a cogenerative fractal solar collector

(a) is a general view of the collector, (b) is a view of the collector half-ring photo of the placement of solar panels c-CFSC 1-tank. 2-the groove from which the hot water comes out. 3-photo panel. 4-1 fractal. 5-2 fractal. 6-3 fractal. 7- Flexible solar photo panel

The main advantages of cogenerative fractal solar collectors are: high process efficiency even at temperatures below zero; ease of installation of the entire structure; resistance of the collector to wind; duration of operation. In the CFSC, the aperture area is located in the space of a parabolic concentrator.

Thus, the aperture region serves as a second source of solar energy for fractally arranged absorbers, that is, the sun's rays passing through the fractally arranged absorbers are reflected from the diaphragm region and heat the fractally arranged toroidal absorbers for the second time.

To achieve the expected result, the parabolic shape of the concentrator, fractal toroidal evacuated absorbers are installed in a parabolic space, so that each absorber not only has a different temperature of toroidal fractal water, but also is a future fractal secondary source of solar energy. This is shown in Fig. 2.

To determine the energy characteristics of the proposed CFSC, an experiment was conducted. Experimental studies were conducted in Kazakhstan/Turkestan ($68.23^{c}irc$ latitude and $43.3051^{c}irc$ longitude) under climatic conditions as of March 1, 2023 using a three-row toroidal fractal structure.

The experiments were conducted in an environment with an average wind speed of 2.5 m/s, a relative humidity of 45% and a cloud cover of 2/8. Wind speed increases heat loss due to convection on the outer surface.

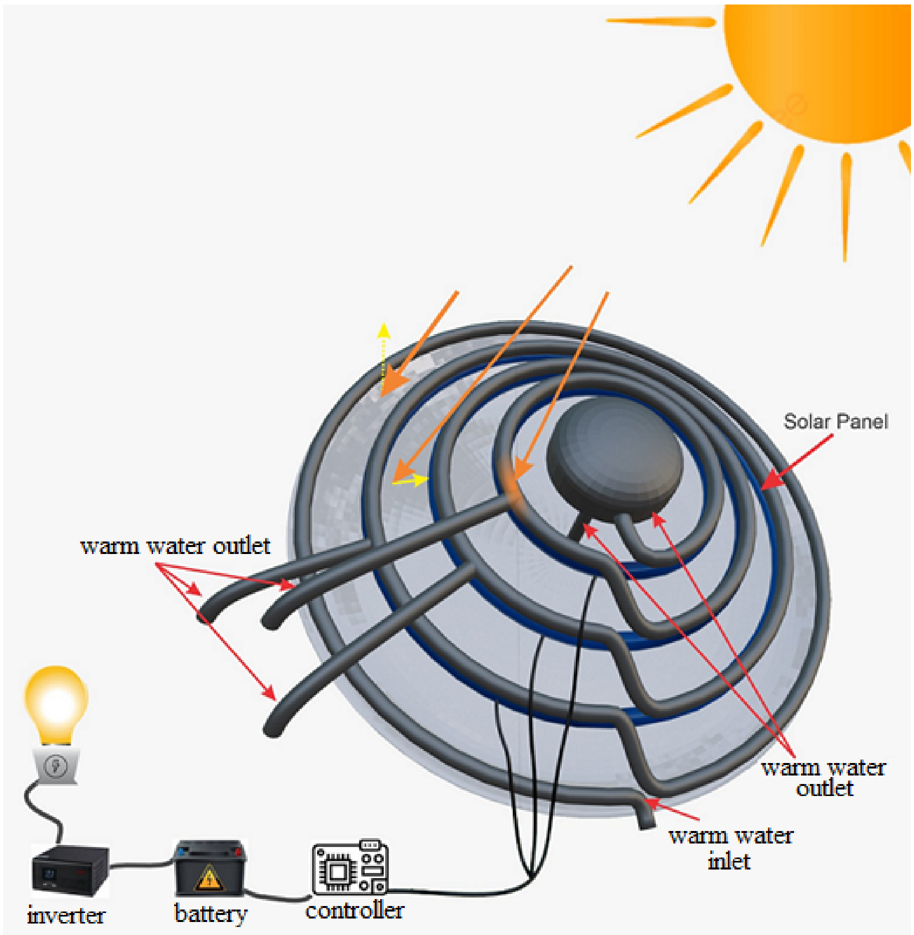

Figure 2 Experimental installation of a cogeneration fractal solar collector

The order of the experiment.

1. First of all, we measure the radius of each fractal located in the solar collector R_1, R_2, R_3 and the inner radius of the fractal tube r.

$$R_1 = 0.15m, \;\; R_2 = 0.30m, \;\; R_3 = 0.40m, \;\; r = 0.016m.$$

2. Using the measured radii, we calculate the surface area of each fractal pipeline A_{a1}, A_{a2}, A_{a3}.

$$A_{a_1} = 0.13\,\mathrm{m}^2, \quad A_{a_2} = 0.19\,\mathrm{m}^2, \quad A_{a_3} = 0.25\,\mathrm{m}^2.$$

3. The hourly values of the useful part of the solar radiation power are found by the formula, $F_R = 0.9$, for each fractal.

$$Q_{l_{a1}}^{hour} = AF_R\left(I_{hour}^{abs} - \gamma_{hour}^{\text{TП}}\right),$$

where A – each area of the absorber

F_R – the coefficient of heat transfer from the solar collector

$I_{hour}^{absorbed}$ – corresponding hourly values of absorbed solar radiation

$\gamma_{hour}^{\text{TП}}$ – hourly value of heat losses

For the first absorber

$$Q_{l_{a1}}^{12^{00}} = A_{a_1} F_R\left(I_{12^{00}}^{\text{abs}} - \gamma_{12^{00}}^{\text{TП}}\right) = 0.13 \cdot 0.9\,(704.8 - 300) = 47.4\,\mathrm{W/m}^2.$$

For the second absorber

$$Q_{l_{a2}}^{12^{00}} = A_{a_2} F_R \left(I_{12^{00}}^{\text{abs}} - \gamma_{12^{00}}^{\text{TП}}\right) = 0.19 \cdot 0.9\,(704.8 - 300) = 69.2\,\text{W/m}^2.$$

For the third absorber

$$Q_{l_{a3}}^{12^{00}} = A_{a_3} F_R \left(I_{12^{00}}^{\text{abs}} - \gamma_{12^{00}}^{\text{TП}}\right) = 0.25 \cdot 0.9\,(704.8 - 300) = 91.1\,\text{W/m}^2.$$

4. Determine the area of the solar panel under each absorber $A_1,\ A_2,\ A_3$
5. We determine the area of the solar panel of each absorber
R_1 – the first radius of the fractal (0.15 m)

$$A_1 = \pi R_1^2 = 3.14 \cdot (0.15)^2 = 0.07\,\text{m}^2.$$

The area of the solar panel under the second absorber is determined by the formula A_2

$$A_2 = l_2 \cdot a.$$

First of all, we measure the length of the second fractal. $R_2 = 0.3\,\text{m}$.

$$l_2 = 2\pi R_2 = 2 \cdot 3.14 \cdot 0.3 = 1.9\,\text{m}.$$

Similarly, we determine the area of the solar panel under the second absorber and the third absorbers. A_2 and A_3.

6. The power of the solar panel is determined by the formula P

$$P = A \cdot \eta \cdot I_{\text{reflection}}.$$

Where:
A - solar panel area (m^2),

η - solar panel efficiency; 15% corresponds to $\eta = 0.15$.

$I_{\text{reflection}}$ – reflected solar radiation power $(W/m)^2$
We find the area of the reflection area of sunlight

$$A_{\text{par}} = \pi \left(h^2 + (2a)^2\right) = 3.14 \left((0.25)^2 + (2 \cdot 0.40)^2\right) = 1.2\,\text{m}^2,$$
$$A_{\text{absor}} = A_1 + A_2 + A_3 = 0.13 + 0.19 + 0.25 = 0.57\,\text{m}^2,$$
$$A_{\text{s.area}} = A_{\text{par}} - A_{\text{absor}} = 1.2 - 0.57 = 0.63\,\text{m}^2.$$

7. Calculate the value of reflected solar radiation from a parabolic concentrator, $F_R = 0.9$, $A_{\text{s.area}} = 0.63\,\text{m}^2$

$$I_{\text{reflection}}^{12^{00}} = A_{\text{s.area}} F_R \left(I_{12^{00}}^{\text{abs}} - \gamma_{12^{00}}^{\text{TП}}\right) = 0.63 \cdot 0.9\,(704.8 - 300) = 229.5\,\text{W/m}^2$$

Doubles the value of solar radiation reflected from a parabolic concentrator 8. We will find the power of the solar panel in each fractal of the CFSC using the formula (9) P_1, P_2, P_3.

$$P_1 = A_1\,\eta\,I_{\text{reflection}}^{12^{00}} = 0.07 \cdot 0.35 \cdot 700 = 17.2\,\text{W},$$
$$P_2 = A_2\,\eta\,I_{\text{reflection}}^{12^{00}} = 0.08 \cdot 0.35 \cdot 700 = 19.6\,\text{W},$$
$$P_3 = A_3\,\eta\,I_{\text{reflection}}^{12^{00}} = 0.10 \cdot 0.35 \cdot 700 = 24.5\,\text{W}.$$

9. We calculate the total power received from the solar panel from CFSC at 12: 00 P_G.

$$P_G = P_1 + P_2 + P_3 = 17.2 + 19.6 + 24.5 = 61.3\,\text{W}.$$

Table 1 CFSC energy characteristics

CFSC energy Description	**Daily watch**							
	10^{00}	11^{00}	12^{00}	13^{00}	14^{00}	15^{00}	16^{00}	17^{00}
$I_{\text{hour}}^{\text{ТП}}$, W/m^2	180	200	250	300	350	300	250	200
R	2.56	2.18	2.09	2.00	2.00	2.09	2.18	2.56
t, ℃	8	9	10	11	12	13	14	13
$I_{\text{hour}}^{\text{НП}}$, W/m^2	460.8	436	522.5	600	700	627	549.4	512
$I_{\text{hour}}^{\text{abs}}$, W/m^2	354.8	335.7	402.3	462	539	482.8	423	394.2
$\gamma_{\text{hour}}^{\text{ТП}}$, W/m^2	312	306	300	294	288	282	276	282
Q_{la1}, W/m^2	5	8.5	21.9	39.6	60.4	49.5	34.2	24.1
Q_{la2}, W/m^2	6.2	10.3	24.7	46.2	78.1	56.9	34.2	26.2
Q_{la3}, W/m^2	7.3	12.0	37.5	50.7	86.0	66.3	36.1	34.2
P_1, W	14.7	15.9	17.2	18.4	19.6	21.0	19.6	17.2
P_2, W	16.8	18.2	19.6	21.0	22.4	23.8	22.4	19.6
P_3, W	21	22.8	24.5	26.3	35.0	29.8	28.0	24.5
P_{gen}, W	52.5	56.9	61.3	65.7	77	74.6	70	61.3
Q_{gen}, W/m^2	71	84	145.4	202.2	301.5	247.3	174.5	145.8
η_{hour}^{G}, %	60	70	74	75.4	76	75.3	75	74.6

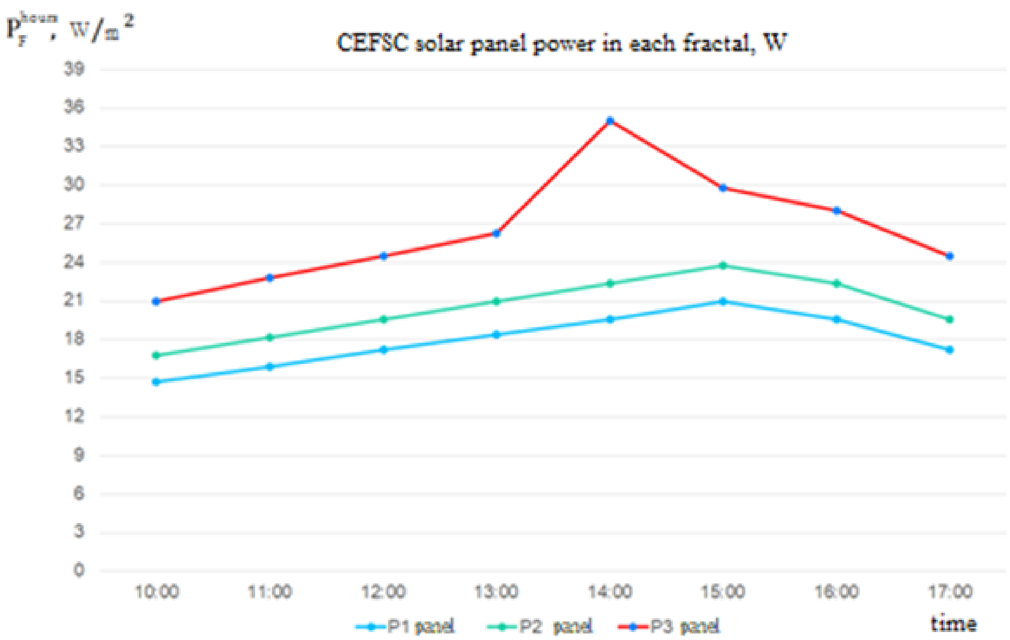

Figure 3 Graph of the electric power on the solar panel in each fractal of the CFSC by daylight hours

As an example below, we will calculate the efficiency efficiency for CFSC in hours 12^{00} For this, we have selected the following initial values from Table 4.

$$I_{12^{00}}^{\text{НП}} = 555.5\,\text{W/m}^2, \quad Q_{\text{gen}}^{12^{00}} = 196.5\,\text{W/m}^2, \quad P_{\text{gen}}^{12^{00}} = 17.25\,\text{W}.$$

Efficiency for the CFSC as a whole, where we determine the total area of the solar panel $A_{\text{s.area}} = 0.63\,\text{m}^2$.

$$\eta_{12^{00}}^{G} = \frac{Q_{\text{gen}}^{12^{00}} + P_{\text{gen}}^{12^{00}}}{A_{\text{gen}}\, I_{12^{00}}^{\text{НП}}} \times 100 = \frac{196.5 + 61.3}{0.63 \cdot 555.5} \times 100 = 74\%.$$

Taking into account the corresponding hourly electrical values of CEFSC $Q_{\text{gen}}^{\text{hour}}$ and $I_{\text{hour}}^{\text{НП}}$, their daily (total) values are determined by the formulas $\sum Q_{\text{gen}}^{\text{hour}}$ and $\sum I_{\text{hour}}^{\text{НП}}$, using formula (9) we found the daily efficiency of CFSC:

$$\eta_{\text{FSC, day}} = \frac{\sum Q_{\text{gen}}^{\text{hour}}}{A_{\text{absor}} \sum I_{\text{hour}}^{\text{НП}}} \times 100 =$$
$$= \frac{71 + 84 + 145.4 + 202.2 + 301.5 + 247.3 + 174.5 + 145.8}{0.63 \cdot (460.8 + 436 + 522.5 + 600 + 700 + 627 + 549.4 + 512)} \times 100 =$$
$$= \frac{1371.7}{4407.7 \cdot 0.48} \times 100 = 71.3\%.$$

Using the following empirical formula in the CEFSC, we determined the average daily value of efficiency:

$$\eta_{\text{VFSC, day}}^{\text{aver}} = \frac{\sum \eta_{\text{gen}}}{n} = \frac{71 + 84 + 145.4 + 202.2 + 301.5 + 247.3 + 174.5 + 145.8}{24} = 58.3\%.$$

where $\eta_{\text{gen}} = \sum \eta_{\text{hour}}$, – that is, the hourly efficiency values in CEFSC taken from formula (7).

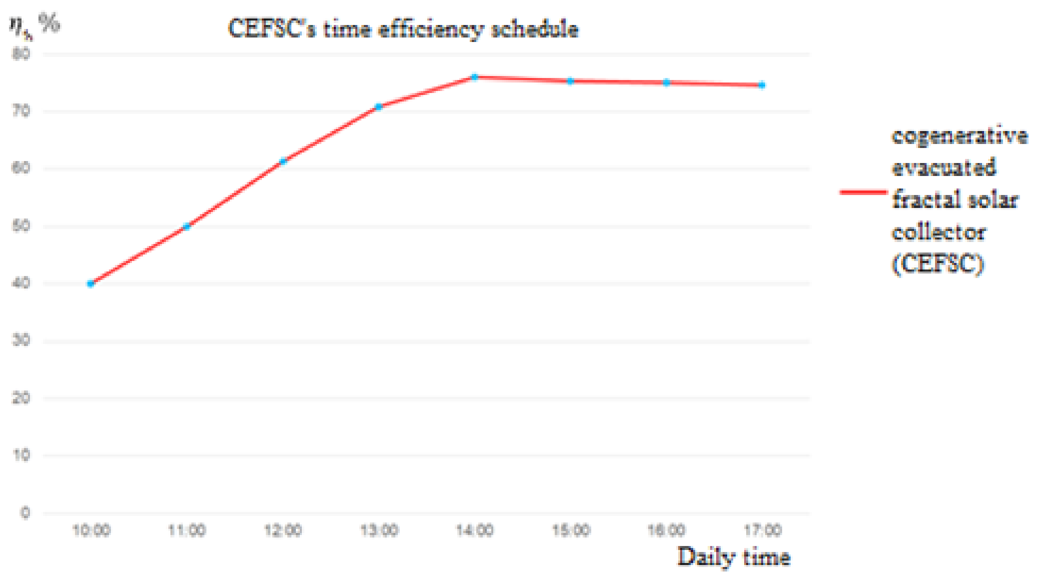

Figure 4 The efficiency graph calculated according to the daily hours of the CFSC

This graph shows the dynamics of the increase in the efficiency of CFSC. This increase in efficiency is explained by the operation of the installation in the cogeneration mode.

3 Conclusion

The effective use of solar collectors largely depends on how we use solar insolation. Our research has shown that by changing the design of the absorbers in the aperture area, the use of solar insolation in a solar installation can increase its efficiency. The efficiency of installations performed with this design can reach 80-85%. The new type of solar collector described in this paper is called a cogenerative fractal solar collector (CFSC). On the other hand, the creation of a CFSC, producing both heat and electricity at the same time, will stimulate the further development of solar collectors.

Based on the results of the experiment, it can be said that CFSC has shown itself to be on the good side in terms of the simplicity of its design and the low cost of consumable material, the small footprint, and the efficiency of using solar radiation. Summarizing the work, we can say the following:

1. Absorbers and their locations in solar collectors are one of the main design elements in the heat supply of solar installations, on which both the energy and economic performance of solar heating systems depend.
2. In flat solar collectors, the absorbers are located in a planar projection. In such structures, the reflected from the aperture area is not used to generate thermal energy.
3. When the aperture area is a parabolic plane and the absorbers are arranged fractally in this plane according to the principle of the Fibonacci number, then solar insolation can be used multiple times.
4. The concept proposed in the work of methods to increase the efficiency of using solar insolation by changing the location of absorbers in the aperture area of water heating collectors is objective.
5. The results of the experiments have shown that changing the location of absorbers in the aperture area can increase the efficiency of work FSC.

References

[1] Avezova N.R., Avezov R.R., Rustamov N.T., Vakhidov A., Suleymanov Sh.I. Resource indexes of flat solar water-heating collectors in hot-water supply systems: 4. Specific collector thermal yield and efficiency // *Journal of Applied Solar Energy*. – 2013. – Vol. 49, Issue 4. – P. 202-210.

[2] Rustamov N.T., Meirbekov A.T., Meirbekov B.K. Power balance equation of a fractal solar collector // *Solar Engineering*. – 2017. – № 4. – C. 38-42.

[3] Rustamov N.T., Meirbekov B.K., Mukhamedzhanov N. Determination of the efficiency of a fractal collector // *Series "Physical and Mathematical Sciences"*. – Almaty, 2017. – № 3(59). – P. 194-198.

[4] Rustamov N.T., Kibishov A.T., Sukhanov M.Ya. Some features of a fractal solar collector // *Scientific Journal "Archivarius"*. – 2021. – Vol. 7, Issue 1(55). – P. 36-42.

[5] Kibishov A., Kilic G.A., Rustamov N., Genc N. Thermal analysis of radiation heat transfer of improved fractal solar collectors // *Applied Sciences*. – 2024. – Vol. 14, Issue 23. – Art. 11155. – doi: `http://dx.doi.org/10.3390/app142311155`.

[6] Rustamov N.T., Kibishov A.T., Isroilov F.M., Yernazar K.E. Efficiency of a vacuum fractal solar collector // *European Journal of Technical and Natural Sciences*. – 2023. – № 2. – P. 46-53.

[7] Rustamov N.T., Meirbekov A.T., Kibishov A.T. *Vacuum solar collector. – Patent of the Republic of Kazakhstan for inventions*. – 2023. – No. 36213 (30.06.2023).

[8] Rustamov N., Kibishov A., Genc N., Babakhan Sh., Kamal E. Thermal conductivity of a vacuum fractal solar collector // *International Journal of Renewable Energy Research*. – 2023. – Vol. 13, Issue 2. – P. 613-618.

[9] Rustamov N.T., Kibishov A.T., Mukhamejanov N.B. Thermal and energy characteristics of a cogenerative fractal solar collector // *European Journal of Engineering and Natural Sciences*. – 2024. – № 1. – P. 3-10. doi: http://dx.doi.org/10.29013/EJTNS-24-1-3-10.

[10] Rustamov N., Meirbekova O., Kibishov A., Babakhan S., Berguzinov A. Creation of a hybrid power plant operating on the basis of a gas turbine engine // *Eastern-European Journal of Enterprise Technologies*. – 2022. – № 2/8(116). doi: http://dx.doi.org/10.15587/1729-4061.2022.255451.

[11] Rustamov N.T., Kibirov A.T., Shertailakov G.M., Isroilov F.M. Efficiency of fractal solar collectors // *Proc. International Scientific and Technical Conference "Innovative solutions to technical, engineering and technological problems of production"*. – Jizzakh, 2022. – P. 699-702.

[12] Rustamov N., Babakhan S., Genc N., Meirbekova O. An improved hybrid wind power plant for small power generation // *International Journal of Renewable Energy Research*. – 2023. – Vol. 13, Issue 2. – P. 629-635. doi: http://dx.doi.org/10.20508/IJRER.V13I2.14193.G8735.

[13] El-Sebaey M.S., Mousavi S.M., Sathyamurthy R., Panchal H., Essa F.A. A detailed review of various design and operating parameters affecting the thermal performance augmentation of flat-plate solar collectors // *International Journal of Ambient Energy*. – 2024. – Vol. 45. – Art. 2351100.

[14] Borzuei D., Moosavian S.F., Ahmadi A., Ahmadi R., Bagherzadeh K. An experimental and analytical study of influential parameters of parabolic trough solar collector // *J. Renew. Energy Environ*. – 2021. – Vol. 8. – P. 52-66.

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КОНСТРУКЦИЯ И ПРИНЦИП РАБОТЫ КОГЕНЕРАТИВНОГО ФРАКТАЛЬНОГО СОЛНЕЧНОГО КОЛЛЕКТОРА

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В работе впервые предлагается фрактальный солнечный коллектор, работающий в режиме когенерации. В данном контексте речь идет о современной инновационной системе, которая использует фрактальные структуры для оптимизации процессов сбора, распределения и преобразования солнечной энергии одновременно в тепловую и электрическую. Приводится детализированная конструкция и результаты проведенного эксперимента данной системы. Показаны этапы проведения эксперимента с целью более глубокого понимания принципов работы коллектора, функционирующего в режиме когенерации. Экспериментально доказана высокая эффективность и перспективность применения данной системы в области возобновляемой энергетики. Кроме того, полученные результаты демонстрируют возможность использования предложенного фрактального коллектора в качестве альтернативного источника энергии, что открывает новые пути развития энергоэффективных и экологически чистых технологий будущего

Ключевые слова: солнечный коллектор, фрактал, когенерация, преобразования, солнечная энергия, генерация энергии, режим когенерации.

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