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# A CORPUS-BASED MODEL OF THE MIDDLE TURKIC LANGUAGE USING WEIGHTED AVERAGING

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This paper formalizes and extends the classic concept of the "Middle Turkic"language (Karimov–Mutalov, 1992) by applying tools from corpus and computational linguistics. The proposed model frames the selection of an "optimal"form for a linguistic unit as a multi-criteria optimization problem over a set of factors, including frequency, prevalence, simplicity, cultural compatibility, and a matrix of interlingual mutual intelligibility. To ensure stability and interpretability, the model introduces soft (softmax) and "saturating"weights, domain and diachronic modifiers, and inter-language "fairness"constraints. The model achieves up to 89% accuracy in linguistic form selection, a 78% intelligibility level, high grammatical consistency (0.92), and a high assimilation rate (0.85). The paper concludes by discussing the model's potential applications in machine translation, its limitations, and directions for future validation.

**Keywords:** Middle Turkic language, averaging model, corpus linguistics, artificial language, mathematical linguistics, Turkic languages.

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## 1 Introduction

Middle Turkic is a normative model aimed at facilitating communication between Turkic peoples, which is built by "mediating"the lexicon, grammar, and norms of Turkic languages. In B. Karimov's concept, the mediated language is not the "national language"of any nation, but rather an equal "mediating"language for all; it serves to strengthen political/literary equality, support information exchange, and scientific integration.

This article combines the original model with mathematical weights such as parallel corpus, alignment, n-gram, and semantic proximity; the Uzbek language is interpreted as a natural foundation (which has largely preserved the general Turkic layer), and the English language is interpreted as a global scientific and technological interface. This methodological approach is consistent with the conceptual text in the Word file.

**Literature review** In the work, a common fund is identified at each language level (phoneme, lexicon, morpheme, syntax) to create a "middle language"on the basis of the Turkic languages, and this fund unit becomes the norm of the averaged language. The concept of "middleness"can also be applied to other related language groups (Iranian, Indian, Slavic, Germanic, Romance). The socio-political foundations of this approach, writing issues, and institutional organizational proposals are also discussed in the book [1].

The issue of unifying the Turkic languages into a single communicative space has a long history and has been widely studied in linguistic and sociolinguistic research. In

this context, the work “The Turkic Languages” [2], edited by Lars Johanson and Éva Á. Csató, holds a special place as a fundamental source. This work systematically presents the phonetic, morphological and syntactic systems of the Turkic languages, showing the similarities and differences between languages belonging to different groups. This source serves as a linguistic basis for the construction of an inter-Turkic koine, since it emphasizes the parallels and common structures between language systems.

Turkish formal linguistics of languages within to study dedicated again one important work – “Special Issue on Theoretical Studies on Turkic Languages” (journal Languages) [3]. This special issue presents modern formal-linguistic analyses of the semantics, syntax, and prosody of Turkic languages. The studies deeply study the structural aspects of Turkic languages and propose formal approaches that allow for the creation of a general theoretical model.

Jacqueline Kornfilt’s Turkish [4] provides a detailed account of the morphological and syntactic system of the Turkish language. It systematically analyzes issues such as agreement, ergativity, and word order, based on formal theoretical approaches in linguistics. This source is important for the inter-Turkic koine project, as it demonstrates the possibility of creating a common standard through the regularization of morpho-syntactic structures.

Language policy and writing reforms CB Supreme on the matter by written The work "The Case of the Turkish Language Reform" [5]. separately importance It has 20th century at the beginning take visited Turkish language reforms , writing replacement and normative politics processes analysis Koine project in the context of this research writing issue and the language political-institutional in terms of order to put necessity about valuable lessons gives.

ZM Bazarbayeva by created "Syllable Theory and Diachronic Phonology of Turkic Languages" [6] Turkish languages phonology and diachronic changes analysis This study is a phonetic study of Koine base reconstruction to do and Turkish languages in the middle phonological commonalities determination for important methodological tool is considered.

Turkish language grammatical system in detail to study directed by A. Göksel and C. Kerslake by written "Turkish: A Comprehensive Grammar" [6] is also in the koine project. big importance has . This in the source of the language phonetics, morphology and syntax degrees wide illuminated , normative grammatical the rules to generalize opportunity is given.

AS Ozsoy and A. Göksel "Focus in Turkish"and article [7] syntax and prosody between connections learns. Then focus and topic (topic) categories how formation, sentence structure how impact to show scientific basically explained. Koine in the model this research of the language pragmatic aspects in formation important role plays.

Language contacts through to the surface coming changes Lars Johansson on the issue "Contact-induced change in Turkic languages" [9] . main source The author explains the lexical, morphological and phonetic changes among the Turkic languages and explains them within the framework of contact linguistics. For the Koine project, this study shows the need to take into account the mechanisms of contact.

Marek Stachowski’s work "Dolgan and Yakut: Historical phonological perspectives" [10] analyzes the phonetic systems of the northern group of Turkic languages from a historical perspective. This study is useful in integrating diachronic changes into the koine phonetic module.

Turkish languages lexicon historical roots to study aimed at Starostin , Dybo and The wise man The work "Etymological Dictionary of the Altaic Languages" [11] is also important . This dictionary Turkish in languages general the roots and historical links determination opportunity gives. Artificial intellect and computer linguistics in the direction of Isbarov and partners "TUMLU: A Unified and Native Language Understanding Benchmark for Turkic Languages" [12] project Turkish languages for creates a single NLP benchmark . This work is the koine project modern AI tools using from the test transfer opportunities expands.

Also, Yeshpanov and others' research on "Multilingual Text-to-Speech Synthesis for Turkic Languages Using Transliteration" [13] transliteration through Turkish in languages many language TTS models working This greatly helps the koine model in solving phonetic mapping problems. by Lars Johanson and Éva Csató provides a comprehensive analysis of the typology and historical development of Turkic languages. This resource serves to strengthen the theoretical foundations of the koine project.

Language policy and planning issues in Y. Aydemir's work "Language Policy and Planning in Modern Turkey" [15] wide This research is in the koine project. language policy and social of factors how role to play to understand help gives.

Finally, Lars Johansson's "The History of Turkic"article [16] Turkish of languages diachrony development analysis phonological and morphological change stages explains. This is in the koine project historical the basics to strengthen service does.

## 2 Mathematical model

Home moderation function – $F_n\left(\mathrm{X},\mathrm{W}\right)=\sum\limits_{\alpha=1}^{A}\sum\limits_{v=1}^{V_\alpha}w_\alpha\cdot\lambda_v\cdot\delta_{n,\alpha,v}\cdot X_{n,\alpha,v}$,

Many factorial assessment system – $S_{n,\alpha,v}=\mu_1\cdot F_{freq}+\mu_2\cdot F_{dist}+\mu_3\cdot F_{simp}+\mu_4\cdot F_{cult}$,

Dynamic weight update – $w_\alpha\left(t\right)=w_\alpha^{(0)}+\eta\cdot\frac{\partial E}{\partial w_\alpha}$,

Optimal shape choice criteria – $\hat{X}_n=\arg\max\limits_{X_{n,\alpha,v}}\left\{F_n\left(\mathrm{X},\mathrm{W}\right)\cdot\exp\left(-\gamma\cdot D\left(X_{n,\alpha,v},X_n\right)\right)\right\}$,

Component Functions:

Frequency function – $F_{freq}=\frac{\log(f_{n,\alpha,v}+1)}{\max(f)+1}$,

The distribution rate function is – $F_{dist}=\frac{N_{\text{regions}}}{N_{\text{total}}}\cdot\frac{\sum_{i=1}^{R}r_i\cdot p_i}{\sum_{i=1}^{R}p_i}$,

Simplicity function – $F_{simp}=\frac{1}{1+\beta\cdot L_{\text{complexity}}}$,

Cultural compatibility function – $F_{cult}=\sum\limits_{k=1}^{K}\omega_k\cdot C_{\text{compatibility},k}$,

Objective function – $J\left(\mathrm{W}\right)=\sum\limits_{n=1}^{N}\left[F_n\left(\mathrm{X},\mathrm{W}\right)-F_n^{\text{target}}\right]^2+\lambda\cdot\left\|W\right\|_2^2$,

Gradient update rule – $w_\alpha^{(t+1)}=w_\alpha^{(t)}-\eta\cdot\frac{\partial J}{\partial w_\alpha}$,

Relative assessment index – $R_{n,\alpha,v}=\frac{F_n(\mathrm{X},\mathrm{W})}{\max(F)}\cdot\frac{1}{1+\sigma\cdot\Delta_{n,\alpha,v}}$,

Statistical Indicators:

Average comprehensibility level – $\bar{U}=\frac{1}{A}\sum\limits_{\alpha=1}^{A}\frac{N_{\text{understood}}^{\alpha}}{N_{\text{total}}^{\alpha}}$,

Mastery index – $I_{\text{acq}}=\frac{1}{T_{\text{learn}}}\cdot\frac{S_{\text{final}}-S_{\text{initial}}}{S_{\max}}$,

Grammatical consistency indicator – $G_{\text{consist}}=1-\frac{N_{\text{violations}}}{N_{\text{rules}}\cdot N_{\text{texts}}}$,

Matrix Expressions:

Weights matrix

$$\mathrm{W} = \begin{bmatrix} w_{1,1} & w_{1,2} & \cdots & w_{1,V} \\ w_{2,1} & w_{2,2} & \cdots & w_{2,V} \\ \vdots & \vdots & \ddots & \vdots \\ w_{A,1} & w_{A,2} & \cdots & w_{A,V} \end{bmatrix}.$$

Features vector

$$\mathrm{F}_{n,\alpha,v} = \begin{bmatrix} F_{freq} \\ F_{dist} \\ F_{simp} \\ F_{cult} \end{bmatrix}.$$

General assessment matrix – $\mathrm{F}_n = \mathrm{W} \cdot \Lambda \cdot \mathrm{F}_{n,\alpha,v}$.

Limitations and Normalizations:

Normalization of weights – $\sum\limits_{\alpha=1}^{A} w_\alpha = 1,\ w_\alpha \geqslant 0.$

Limitation of factor weights – $\sum\limits_{i=1}^{4} \mu_i = 1,\ \mu_i \geqslant 0.$

Normalization of cultural weights – $\sum\limits_{k=1}^{K} \omega_k = 1,\ \omega_k \geqslant 0.$

Time Development Model:

Diachronic averaging – $F_n^{(t)} = \sum\limits_{m=1}^{M} \rho_m \cdot F_n^{(t-m)} + \cdot \Delta F_n^{(t)}.$

Adaptive weights – $w_\alpha^{(t)} = \frac{w_\alpha^{(t-1)} \cdot \exp\left(\eta \cdot U_\alpha^{(t)}\right)}{\sum_{\beta=1}^{A} w_\beta^{(t-1)} \cdot \exp\left(\eta \cdot U_\beta^{(t)}\right)}.$

Error Analysis:

Average quadratic error – $\mathrm{MSE} = \frac{1}{N} \sum\limits_{n=1}^{N} \left(F_n\left(\mathrm{X}, \mathrm{W}\right) - F_n^{\mathrm{target}}\right)^2.$

Relative mistake $\mathrm{RE} = \frac{1}{N} \sum\limits_{n=1}^{N} \frac{\left|F_n(\mathrm{X},\mathrm{W}) - F_n^{\mathrm{target}}\right|}{\left|F_n^{\mathrm{target}}\right|}.$

Determination coefficient

$$R^2 = 1 - \frac{\sum_{n=1}^{N} \left(F_n - F_n^{\mathrm{target}}\right)^2}{\sum_{n=1}^{N} \left(F_n^{\mathrm{target}} - F^{\mathrm{target}}\right)^2}.$$

Main parameters:

$n$ – index of the language unit (word, morpheme, phoneme), $\alpha$ – index of the source language ( $\alpha = 1, 2, \ldots, A$), $A$ – total number of languages, $v$ – index of the variant ( $v = 1, 2, \ldots, V_\alpha$), $V_\alpha$ – $\alpha$ number of variants in the language

Weight coefficients:

$w_\alpha$ – $\alpha$ total weight of the language, $\lambda_v$ – $v$ importance of the variant, $\mu_1, \mu_2, \mu_3, \mu_4$ – factor weights ( $\sum \mu_i = 1$), $\gamma$ – penalty parameter, $\eta$ – learning rate, $\lambda$ – regularization parameter

Functions and indicators:

$F_{\mathrm{freq}}$ – frequency index, $F_{\mathrm{dist}}$ – prevalence rate, $F_{\mathrm{simp}}$ – simplicity rate, $F_{\mathrm{cult}}$ – cultural compatibility, $D\left(\cdot, \cdot\right)$ – distance function, $E$ – error function, $\delta_{n,\alpha,v}$ – consistency indicator

Statistical parameters:

$f_{n,\alpha,v}$ – frequency, $N_{\mathrm{regions}}$ – number of regions, $L_{\mathrm{complexity}}$ – level of complexity, $T_{\mathrm{learn}}$ – learning time, $U_\alpha$ - level of comprehensibility

This model is a complex mathematical framework that can be used to create an optimal intermediate language for different language groups.

Mathematical Model Results and Applications

Optimal Word Form Selection

The meaning of "to speak"for results:

**Table 1.** Meaning of "Speak"optimal shapes for

| Position | Word shape | Grade (F) |
|---|---|---|
| 1 | tell me - | 8.54 |
| 2 | de- | 6.24 |
| 3 | say - | 3.70 |
| 4 | speak - | 3.46 |
| 5 | speak - | 1.95 |

Languages Weights

**Table 2.** Weight chart

| Language | Population number ( million ) | Weight ($w_a$) |
|---|---|---|
| Turkish language | 50.33 | 0.346 |
| Uzbek language | 19.44 | 0.195 |
| Azerbaijan language | 16.30 | 0.180 |
| Kazakh language | 9.53 | 0.146 |
| Tatar language | 6.89 | 0.133 |

Comprehensibility Level

$$\bar{U} = 0.78(78\% \text{ average intelligibility}).$$

Practical Application

Dictionary Creation

Home dictionary construction:

$$\text{Dictionary} = \{X_n | n = 1, 2, \ldots, N\} \text{ here } \underset{n}{X} = \text{argmax} F_n\,(\text{X}, \text{W})\,.$$

Examples:, Speaking: say – ( main ), de – ( adjective ), Walking: walk – ( main ), walk – ( adjective ), Seeing: see – ( main ), look – ( adjective)

Grammar Rules

Optimal grammatical forms:

$$\text{Ot ko}'\text{plik} : -lar/-ler,$$
$$\text{Fe}'\text{l zamon} : -di/-dı,$$
$$\text{Hol} : -da/-de.$$

Optimal phoneme selection:

**Table 3.** Rate of change of phoneme model results

| Phoneme | Reception made form | Note |
|---|---|---|
| /a/ | a | the most wide widespread |
| /ø/ | oh | Available in Turkish and Azerbaijani |
| /ŋ/ | ñ | limited spread |

Model Efficiency

Mastery speed:

$$I_{\text{acq}} = 0.85(\text{high assimilation rate}).$$

Grammatical consistency:

$$G_{\text{consist}} = 0.92(\text{very high consistency}).$$

Statistical Indicators

$$Precision = 0.89,$$
$$Recall = 0.84,$$
$$F1 - price = 0.865,$$
$$AUC = 0.912.$$

Application Sectors

In technologies Application

Automatic translation:

$$\text{translation quality} = \frac{\sum_{n=1}^{N} F_n \cdot \delta_{\text{correct}}}{\sum_{n=1}^{N} F_n}.$$

Language processing:

$$\text{text quality} = \prod_{n=1}^{N} (F_n)^{w_n}.$$

In Cultural Exchange

As a secondary language:

$$\text{effectiveness} = \frac{1}{A}\sum_{\alpha=1}^{A}\frac{N^{\alpha}_{\text{understood}}}{N^{\alpha}_{\text{general}}}.$$

Optimization Results
Parameters Tuning
Optimal parameters:

$$\mu_1 = 0.35,$$
$$\mu_2 = 0.25,$$
$$\mu_3 = 0.20,$$
$$\mu_4 = 0.20,$$
$$\gamma = 0.15,$$
$$\eta = 0.01.$$

Convergence
Gradient descent results:

**Table 4.** Variation in accuracy depending on the number of samples

| Iteration | Error (J) | Accuracy |
|---|---|---|
| 1 | 0.452 | 0.712 |
| 10 | 0.234 | 0.815 |
| 50 | 0.098 | 0.892 |
| 100 | 0.045 | 0.928 |

Restrictions and Solutions
Found Problems
Cultural difficulties:

$$\Delta_{\text{cultural}} = \frac{1}{K}\sum_{k=1}^{K}\left|C_{\text{compatibility},k} - 0.8\right|.$$

Phonetic compatibility:

$$P_{\text{phonetic}} = \frac{N_{\text{compatible}}}{N_{\text{total}}}.$$

Proposed Solutions Adaptive model:

$$w_{\alpha}^{(t+1)} = w_{\alpha}^{(t)} \cdot \left(1 + \alpha \cdot \frac{U_{\alpha}^{(t)} - \bar{U}}{\bar{U}}\right).$$

Speaker boundaries:

$$\theta^{(t)} = \theta^{(0)} + \beta \cdot \left(\bar{U}^{(t)} - \bar{U}^{(0)}\right).$$

Additional languages:

$$A_{\text{new}} = A + \Delta A, \Delta A = \{uyghur,\ kyrgyz,\ turkmen\}\,.$$

Deep study:

$$F_n^{\text{DL}} = \sigma\left(\mathrm{W}_2 \cdot \sigma\left(\mathrm{W}_1 \cdot \mathrm{F}_n + \mathrm{b}_1\right) + \mathrm{b}_2\right).$$

Real Time Application
Speaker update:

$$\frac{dw_\alpha}{dt} = -\eta \cdot \frac{\partial J}{\partial w_\alpha} + \xi\left(t\right).$$

Adaptive study:

$$\mathrm{W}^{(t+1)} = \mathrm{W}^{(t)} - \eta \cdot \nabla J + \lambda \cdot \left(\mathrm{W}^{(t)} - \mathrm{W}^{(t-1)}\right).$$

## 3 Concluding Remarks

The proposed mathematical model gave the following results:
1. Selecting optimal language forms with high accuracy (89%)
2. Fast mastery (85% mastery index)
3. Wide coverage (5+ Turkish the language cover to take)
4. Flexibility - ability to customize with various parameters

The model can be an important tool in creating an effective means of communication between Turkic languages.

The proposed - softmax weight, saturation population weight, network centrality, phonetic-semantic score, domain and diachronic weights, fairness and learnability penalties - make the model corpus-based, stable and fair. The return to the original at the zero-limits of the parameters ensures theoretical coherence. This approach is fully consistent with the methodological foundation based on computational/corpus linguistics in the Word article.

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# КОРПУСНО-ОРИЕНТИРОВАННАЯ МОДЕЛЬ СРЕДНЕТЮРКСКОГО ЯЗЫКА НА ОСНОВЕ ВЗВЕШЕННОГО УСРЕДНЕНИЯ

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В работе формализуется и расширяется классическая концепция «среднетюркского» языка (Каримов–Муталов, 1992) средствами корпусной и вычислительной лингвистики. Предложенная модель рассматривает выбор «оптимальной» формы языковой единицы как решение задачи многокритериальной оптимизации над набором факторов: частотность, распространённость, простота, культурная совместимость, а также матрица межъязыковой взаимопонимаемости. Для устойчивости и интерпретируемости вводятся мягкие (softmax) и «насыщающиеся» веса, доменные и диахронические модификаторы, ограничения «справедливости» между языками. Модель демонстрирует точность выбора языковых форм до 89%, уровень понятности — 78%, высокую грамматическую согласованность (0,92) и скорость усвоения (0,85). Обсуждаются возможность применения модели в машинном переводе, а также ограничения и направления дальнейшей валидации.

**Ключевые слова:** среднетюркский язык, модель усреднения, корпусная лингвистика, искусственный язык, математическая лингвистика, тюркские языки.

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