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OPTIMIZATION OF APPROXIMATE INTEGRATION FORMULAS FOR PERIODIC FUNCTION CLASSES

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This study explores the optimization of quadrature formulas for the approximate integration of periodic functions within a specific functional space. The research focuses on developing optimal quadrature formulas by deriving an analytical expression for the error associated with the integration process. By employing Fourier transform techniques and the concept of an extremal function, the study establishes a precise representation of the error. Additionally, optimal coefficients for the quadrature formula are determined to minimize this error, yielding an explicit solution. The results demonstrate enhanced accuracy compared to existing approaches, with the error characterized through a series expansion that reveals its asymptotic behavior. These findings advance the efficiency of numerical integration for periodic functions, offering potential applications in mathematical analysis, scientific computing, and related disciplines.

Keywords: optimal quadrature formula, Hilbert space, the error functional, Fourier transform.

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1 Introduction

In the future, there is a need to further optimize quadrature formulas for periodic functions and develop new methods. With the help of modern computing technologies and algorithms, it is expected that faster and more accurate integral calculations will become possible. The currently available optimal quadrature formulas are constantly evolving, taking into account technological and scientific achievements. All this will serve to make mathematical and numerical calculations more efficient and to further improve the applications used in various fields. Thus, the development of optimal quadrature formulas for the integration of periodic functions plays an important role in mathematical analysis, scientific research, and solving real-world problems. Research and innovation in this area will drive the development of future computing technologies, as well as create opportunities for more efficient and accurate calculations in various fields.

We will consider the construction of the optimal quadrature formula in Sobolev space $\widetilde{W}_2^{(m,m-1,m-2)}(0,1)$.

Here, the space $\widetilde{W}_2^{(m,m-1,m-2)}(0,1)$ is the Sobolev of periodic functions, where the norm of the function is introduced by the following scalar product

$$\langle \varphi, \psi \rangle_{\widetilde{W}_2^{(m,m-1,m-2)}(0,1)} =$$

$$= \int_0^1 \left(\varphi^{(m)}(x)\psi^{(m)}(x) + 2\varphi^{(m-1)}(x)\psi^{(m-1)}(x) + \varphi^{(m-2)}(x)\psi^{(m-2)}(x)\right) dx, \quad (1)$$

$$\|\varphi\|_{\widetilde{W}_2^{(m,m-1,m-2)}(0,1)} = \left(\int_0^1 \left(\varphi^{(m)}(x)\right)^2 + 2\left(\varphi^{(m-1)}(x)\right)^2 + \left(\varphi^{(m-2)}(x)\right)^2\right)^{1/2},$$

where the derivative is the generalized derivative.
We enter the following quadrature formula

$$\int_0^1 \varphi(x)dx \approx \sum_{k=1}^{N} C_k \varphi(hk), \quad (2)$$

with the error

$$(l, \varphi) = \int_0^1 \varphi(x)dx - \sum_{k=1}^{N} C_k \varphi(hk), \quad (3)$$

where

$$(l, \varphi) = \int_{-\infty}^{\infty} l(x)\varphi(x)dx,$$

and the corresponding error functional is

$$l(x) = \left(\varepsilon_{(0,1]}(x) - \sum_{k=1}^{N} C_k \delta(x - hk)\right) * \varphi_0(x). \quad (4)$$

Here C_k are the coefficients of formula (2), $h = \frac{1}{N}$, $N \in \mathbb{N}$, $\varepsilon_{(0,1]}(x)$ is the indicator of the interval $(0,1]$, δ is the Dirac's delta-function, $\varphi_0(x) = \sum_{\beta=-\infty}^{\infty} \delta(x - \beta)$

(4) for the error functional $m \geqslant 3$, the following is true $(l, 1) = 0$ and $\sum_{k=1}^{N} C_k = 1$ Our goal is to estimate the error of the quadrature formula (3) from above, for which it is sufficient to calculate the norm of the error functional (4) This leads to the solution of the following two problems, and we will first consider this for $m = 2$.

Problem 1 (4) Find the analytical representation of the error functional norm in $\widetilde{W}_2^{(2,1,0)}(0,1)$ space.

Problem 2 Finding the optimal coefficients of the $C_k = \overset{0}{C}_k$ that minimize $\|l\|$.

2 Find the analytical representation of the error functional norm in $\widetilde{W}_2^{(2,1,0)}(0,1)$ space.

To solve problem 1, we use the concept of an extremal function introduced by Sobolev. Using Riesz's theorem for the space $\widetilde{W}_2^{(2,1,0)}(0,1)$, we can write the following

$$(l, \varphi) = \langle \psi_l, \varphi \rangle_{\widetilde{W}_2^{(2,1,0)}(0,1)} = \int_0^1 \varphi''(x)\psi''(x)dx + 2\int_0^1 \varphi'(x)\psi'(x)dx + \int_0^1 \varphi(x)\psi(x)dx =$$

$$= \int_0^1 \left(\psi^{(4)}(x) - 2\psi^{(2)}(x) + \psi(x)\right) \varphi(x)dx.$$

From the above equation we get the following equation

$$\psi_l^{(4)}(x) - 2\psi_l^{(2)}(x) + \psi_l(x) = l(x). \tag{5}$$

Theorem 1. In the Sobolev space of $\widetilde{W}_2^{(2,1,0)}(0,1)$ periodic functions, the extremal function of the quadratic formula (2) has the following form

$$\psi_l(x) = 1 - \sum_{k=1}^{N} C_k \sum_{\beta=-\infty}^{\infty} \frac{e^{-2\pi i\beta(x-hk)}}{(2\pi\beta)^4 + 2(2\pi\beta)^2 + 1}. \tag{6}$$

Proof. To find the generalized periodic solution of differential equation (5), we apply the Fourier transform to both sides of the equation and use the following properties of the Fourier transform

$$F[\varphi] = \int_{-\infty}^{\infty} \varphi(x) e^{2\pi i\beta x} dx,$$

$$F^{-1}[\varphi] = \int_{-\infty}^{\infty} \varphi(\beta) e^{-2\pi i\beta x} d\beta,$$

$$F[\varphi^{(\alpha)}] = (-2\pi i\beta)^\alpha F[\varphi],\ (\alpha \in \mathbb{N}),$$

$$F[\varphi * g] = F[\varphi] \cdot F[g],$$

$$F[\varphi \cdot g] = F[\varphi] * F[g],$$

$$F[\varphi_0(x)] = \varphi_0(\beta),$$

$$F^{-1}[F[\varphi(x)]] = \varphi(x).$$

Here $*$ is the convolution operation [1].

We apply Fourier transform to both sides of equation (5)

$$F[\psi_l^{(4)}(x) - 2\psi_l^{(2)}(x) + \psi_l(x)] = F[l(x)].$$

Since, the Fourier transform is linear operator, we have

$$((2\pi p)^2 + 1)^2 F[\psi_l] = F\left[\left(\varepsilon_{(0,1]}(x) - \sum_{k=1}^{N} C_k \delta(x - hk)\right) * \varphi_0(x)\right]$$

or

$$((2\pi p)^2 + 1)^2 F[\psi_l] = F\left[\varepsilon_{(0,1]}(x) - \sum_{k=1}^{N} C_k \delta(x - hk)\right] \cdot \varphi_0(p),$$

where

$$F[\delta(x - hk)] = \int_{-\infty}^{\infty} \delta(x - hk) e^{2\pi i p x} dx = e^{2\pi i p h k},$$

$$F[\psi_l(x)] = \frac{F\left[\varepsilon_{(0,1]}(x) - \sum_{k=1}^{N} C_k e^{2\pi i p h k}\right] \cdot \varphi_0(p)}{((2\pi p)^2 + 1)^2} =$$

$$= \frac{F[\varepsilon_{(0,1]}(x)] \cdot \varphi_0(p)}{((2\pi p)^2+1)^2} - \sum_{k=1}^{N} C_k \frac{e^{2\pi i p h k} \cdot \varphi_0(p)}{((2\pi p)^2+1)^2} =$$

$$= \frac{F\left[\varepsilon_{(0,1]}(x)\right] \cdot \sum_{\beta=-\infty}^{\infty} \delta(p-\beta)}{((2\pi p)^2+1)^2} - \sum_{k=1}^{N} C_k \frac{e^{2\pi i p h k} \cdot \sum_{\beta=-\infty}^{\infty} \delta(p-\beta)}{((2\pi p)^2+1)^2}.$$

Using the property $f(x)\delta(x-a) = f(a)\delta(x-a)$ of delta-function, we have the following

$$F[\psi_l(x)] = F\left[\varepsilon_{(0,1]}(x)\right] \cdot \sum_{\beta=-\infty}^{\infty} \frac{\delta(p-\beta)}{((2\pi\beta)^2+1)^2} - \sum_{k=1}^{N} C_k \sum_{\beta=-\infty}^{\infty} \frac{e^{2\pi i \beta h k}\delta(p-\beta)}{((2\pi\beta)^2+1)^2}.$$

Then, we apply the inverse Fourier transform to the above equality and we obtain the following:

$$F^{-1}[F[\psi_l(x)]] = \varepsilon_{(0,1]}(x) * \sum_{\beta=-\infty}^{\infty} \frac{e^{-2\pi i \beta x}}{((2\pi\beta)^2+1)^2} - \sum_{k=1}^{N} C_k \sum_{\beta=-\infty}^{\infty} \frac{e^{2\pi i \beta h k} \cdot e^{-2\pi i \beta x}}{((2\pi\beta)^2+1)^2}.$$

Hence, we get

$$\psi_l(x) = \varepsilon_{(0,1]}(x) * \sum_{\beta=-\infty}^{\infty} \frac{e^{-2\pi i \beta x}}{((2\pi\beta)^2+1)^2} - \sum_{k=1}^{N} C_k \sum_{\beta=-\infty}^{\infty} \frac{e^{2\pi i \beta h k} \cdot e^{-2\pi i \beta x}}{((2\pi\beta)^2+1)^2} =$$

$$= \varepsilon_{(0,1]}(x) * G_2(x) - \sum_{k=1}^{N} C_k G_2(x-hk) = \int_{-\infty}^{\infty} \varepsilon_{(0,1]}(y) G_2(x-y) dy - \sum_{k=1}^{N} C_k G_2(x-hk) =$$

$$= \int_0^1 G_2(x-y)dy - \sum_{k=1}^{N} C_k G_2(x-hk) = 1 - \sum_{k=1}^{N} C_k G_2(x-hk),$$

where

$$G_2(x) = \sum_{\beta=-\infty}^{\infty} \frac{e^{-2\pi i \beta x}}{(2\pi\beta)^4 + 2(2\pi\beta)^2 + 1}, \tag{7}$$

now we simplify

$$\int_0^1 G_2(x-y)\, dy,$$

$$\int_0^1 G_2(x-y)\, dy = \sum_{\beta=-\infty}^{\infty} \int_0^1 \frac{e^{-2\pi i \beta (x-y)}}{((2\pi\beta)^2+1)^2}\, dy = 1 + \sum_{\beta \neq 0} \frac{e^{-2\pi i \beta x}}{((2\pi\beta)^2+1)^2} dy =$$

$$= 1 + \sum_{\beta \neq 0} \frac{e^{-2\pi i \beta x}}{((2\pi\beta)^2+1)^2} \int_0^1 e^{2\pi i \beta y}\, dy = 1 + \sum_{\beta \neq 0} \frac{e^{-2\pi i \beta x}}{((2\pi\beta)^2+1)^2} \cdot \left. \frac{e^{2\pi i \beta} - 1}{2\pi i \beta} \right|_0^1 =$$

$$= 1 + \sum_{\beta \neq 0} \frac{e^{-2\pi i\beta x}}{((2\pi\beta)^2+1)^2} \cdot \frac{e^{2\pi i\beta}-1}{2\pi i\beta} = 1.$$

And so, theorem 1 is proved from the last equality.

Now, to solve problem 1, we simplify the error functional (4)

$$l(x) = \left(\varepsilon_{[0,1]}(x) - \sum_{k=1}^{N} C_k \delta(x-hk)\right) * \varphi_0(x) =$$

$$= \left(\varepsilon_{[0,1]}(x) - \sum_{k=1}^{N} C_k \delta(x-hk)\right) * \sum_{\beta=-\infty}^{\infty} \delta(p-\beta) =$$

$$= \sum_{\beta=-\infty}^{\infty} \int \varepsilon_{[0,1]}(y) \cdot \delta(x-\beta-y)\,dy - \sum_{k=1}^{N} C_k \sum_{\beta=-\infty}^{\infty} \int \delta(y-hk)\delta(x-\beta-y)\,dy =$$

$$= \sum_{\beta=-\infty}^{\infty} \varepsilon_{[0,1]}(x-\beta) - \sum_{k=1}^{N} C_k \sum_{\beta=-\infty}^{\infty} \delta(x-\beta-hk) = 1 - \sum_{k=1}^{N} C_k \sum_{\beta=-\infty}^{\infty} \delta(x-\beta-hk).$$

We calculate the analytical form of the error functional norm

$$\|l\|^2_{\widetilde{W}_2^{(2,1,0)}(0,1)^*} = \int_0^1 l(x)\psi_l(x)\,dx =$$

$$= \int_0^1 \left(1 - \sum_{k=1}^{N} C_k \sum_{\beta=-\infty}^{\infty} \delta(x-\beta-hk)\right) \left(1 - \sum_{k'=1}^{N} C_{k'} \sum_{\beta=-\infty}^{\infty} \frac{e^{-2\pi i\beta(x-hk')}}{((2\pi\beta)^2+1)^2}\right) dx.$$

From the above equality, we obtain the following

$$\|l\|^2_{\widetilde{W}_2^{(2,1,0)}(0,1)^*} = 1 - \sum_{k=1}^{N} C_k - \sum_{k'=1}^{N} C_{k'} +$$
$$+ \sum_{k=1}^{N}\sum_{k'=1}^{N} C_k C_{k'} + \sum_{k=1}^{N}\sum_{k'=1}^{N} C_k C_{k'} \sum_{\beta \neq 0} \frac{e^{-2\pi i\beta h(k-k')}}{((2\pi\beta)^2+1)^2}. \quad (8)$$

So problem 1 is solved.

3 Coefficients of the optimal quadrature formula (2)

Theorem 2. The coefficients of the quadratura formula in the form (2) that minimizes the norm of the error functional are as follows:

$$C_k = C = \frac{4(2-e^h-e^{-h})}{e^{-h}-e^h-2h}.$$

Proof. Taking the first derivative of the coefficient from equality (8) and equating it to zero, we obtain the following equality

$$\sum_{k'=1}^{N} C_{k'} + \sum_{k'=1}^{N} C_{k'} \sum_{\beta\neq 0} \frac{e^{-2\pi i\beta h(k-k')}}{((2\pi\beta)^2+1)^2} = 1. \quad (9)$$

We use the fact that the coefficients are equal in the class of periodic functions, that is, $C_{k'} = C$

$$N\cdot C + C\cdot \sum_{\beta\neq 0} \frac{e^{-2\pi i\beta hk}}{((2\pi\beta)^2+1)^2} \cdot \sum_{k'=1}^{N} e^{2\pi i\beta hk'} = 1 \quad \text{and} \quad \sum_{k'=1}^{N} e^{2\pi i\beta hk'} = \begin{cases} 0, & \beta h \neq \gamma, \quad \gamma \in \mathbb{Z}, \\ N, & \beta h = \gamma, \quad \gamma \in \mathbb{Z}. \end{cases}$$

$$N\cdot C + C\cdot \sum_{\gamma\neq 0} \frac{e^{-2\pi i\gamma k}}{((2\pi\gamma N)^2+1)^2} \cdot N = 1,$$

$$N\cdot C + N\cdot C\cdot \sum_{\gamma\neq 0} \frac{1}{((2\pi\gamma N)^2+1)^2} = 1. \quad (10)$$

$\sum_{\gamma\neq 0} \frac{1}{((2\pi\gamma N)^2+1)^2}$ is even function that's way we can rewrite

$$\sum_{\gamma\neq 0} \frac{1}{((2\pi\gamma N)^2+1)^2} = 2\cdot \sum_{\gamma=1}^{\infty} \frac{1}{((2\pi\gamma N)^2+1)^2} = \left(\frac{h}{2\pi}\right)^4 \cdot 2\sum_{\gamma=1}^{\infty} \frac{1}{\left(\gamma - \frac{hi}{2\pi}\right)^2 \left(\gamma + \frac{hi}{2\pi}\right)^2}.$$

Based on the method of residue theory [2]

$$f(\gamma) = \frac{1}{\left(\gamma - \frac{hi}{2\pi}\right)^2 \left(\gamma + \frac{hi}{2\pi}\right)^2}; \qquad 2\sum_{\gamma=1}^{\infty} f(\gamma) + f(0) = \sum_{\gamma=-\infty}^{\infty} f(\gamma).$$

$$\sum_{\gamma=\infty}^{\infty} f(\gamma) = -\sum_{z_1,z_2} Res(\pi ctg(\pi z)\cdot f(z)); \quad z_1 = \frac{hi}{2\pi} \quad \text{and} \quad z_2 = -\frac{hi}{2\pi}.$$

Simplifying using the method of residue theory, we can write:

$$\sum_{\gamma=-\infty}^{\infty} f(\gamma) = -\frac{4\pi^4}{h^3} \cdot \frac{e^h - e^{-h} - 2h}{e^h + e^{-h} - 2} \quad \text{else} \quad f(0) = \left(\frac{2\pi}{h}\right)^4,$$

$$2\sum_{\gamma=1}^{\infty} f(\gamma) = \sum_{\gamma=-\infty}^{\infty} f(\gamma) - f(0) = \frac{4\pi^4}{h^3} \cdot \left(\frac{e^h - e^{-h} - 2h}{2 - e^h - e^{-h}} - \frac{4}{h}\right).$$

Taking into account the above equality, the coefficient from equation (10) is equal to:

$$C = \frac{4(2 - e^h - e^{-h})}{e^h - e^{-h} - 2h}. \quad (11)$$

The optimal coefficient, i.e. $\frac{4(2-e^h-e^{-h})}{e^{-h}-e^h-2h}$, tends to 0 at expression $h \to 0$.

$$\lim_{h\to 0} \frac{4(2 - e^h - e^{-h})}{e^{-h} - e^h - 2h} = \lim_{h\to 0} \frac{4(-e^h + e^{-h})}{-e^h - e^{-h} - 2} = 0.$$

The theorem has been proven.

Theorem 3. The square of the norm of the optimal quadratura formula in the space $\widetilde{W}_2^{(2,1,0)}(0,1)$ is expressed as

$$\|l^0\|^2_{\widetilde{W}_2^{(2,1,0)}(0,1)^*} = 1 + \frac{4(e^h + e^{-h} - 2)}{h(e^{-h} - e^h - 2h)},$$

$$\|l^0\|^2 = \frac{h^4}{720} - \frac{h^6}{15120} + O(h^8).$$

Proof. Expression (8) can be written as follows, taking into account the equality of coefficients and expression (9):

$$\|l\|^2_{\widetilde{W}_2^{(2,1,0)}(0,1)^*} = 1 - \sum_{k'=1}^{N} C_{k'} - \sum_{k=1}^{N} C_k + \sum_{k=1}^{N}\sum_{k'=1}^{N} C_k C_{k'} + \sum_{k=1}^{N}\sum_{k'=1}^{N} C_k C_{k'} \sum_{\beta\neq 0} \frac{e^{-2\pi i\beta h(k-k')}}{((2\pi\beta)^2+1)^2} =$$

$$= 1 - 2\sum_{k'=1}^{N} C_{k'} + \sum_{k'=1}^{N} C_{k'} \left(\sum_{k=1}^{N} C_k + \sum_{k=1}^{N} C_k \sum_{\beta\neq 0} \frac{e^{-2\pi i\beta h(k-k')}}{((2\pi\beta)^2+1)^2} \right) =$$

$$= 1 - \sum_{k'=1}^{N} C_{k'} = 1 - NC.$$

Given the determined coefficient and the fact that $N = \frac{1}{h}$, the square of the error functional norm is equal to:

$$\|l^0\|^2_{\widetilde{W}_2^{(2,1,0)}(0,1)^*} = 1 + \frac{4(e^h + e^{-h} - 2)}{h(e^{-h} - e^h - 2h)}, \quad \text{or} \quad \|l^0\|^2 = \frac{h^4}{720} - \frac{h^6}{15120} + O(h^8).$$

The theorem has been proven.

4 Conclusion

In this work, the Sobolev extremum function $\widetilde{W}_2^{(m,m-1,m-2)}(0,1)$ in the space of periodic functions, the analytical form of the square of the error functional norm, and the coefficients are found. It is clearly shown by the series that the square of the error functional norm is significantly smaller than in other works in the space. The limit of the coefficients when h is not desired is calculated.

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ОПТИМИЗАЦИЯ ПРИБЛИЖЁННЫХ ФОРМУЛ ИНТЕГРИРОВАНИЯ ДЛЯ КЛАССОВ ПЕРИОДИЧЕСКИХ ФУНКЦИЙ

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Данное исследование посвящено оптимизации квадратурных формул для приближённого интегрирования периодических функций в определённом функциональном пространстве. Основное внимание уделяется разработке оптимальных квадратурных формул путём получения аналитического выражения для ошибки, связанной с процессом интегрирования. Используя методы преобразования Фурье и концепцию экстремальной функции, исследование устанавливает точное представление ошибки. Кроме того, определяются оптимальные коэффициенты квадратурной формулы, минимизирующие эту ошибку, что приводит к явному решению. Результаты демонстрируют повышенную точность по сравнению с существующими подходами, при этом ошибка характеризуется через разложение в ряд, раскрывающее её асимптотическое поведение. Эти выводы способствуют повышению эффективности численного интегрирования периодических функций и открывают перспективы для применения в математическом анализе, научных вычислениях и смежных областях.

Ключевые слова: оптимальная квадратурная формула, Гильбертово пространство, функционал ошибки, преобразование Фурье.

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HISOBLASH VA AMALIY MATEMATIKA MUAMMOLARI

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